Measurement of the anomalous spin precession frequency in the Muon $g-2$ experiment at Fermilab

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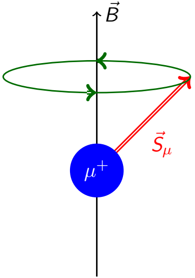
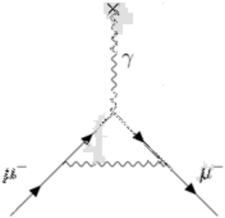
What's the anomalous magnetic moment

- A particle's spin in a magnetic field experiences a torque and a precession motion proportional to its magnetic moment, defined as

$$\vec{\mu} = g\frac{e}{2m}\vec{S}$$

- Prediction from Dirac equation: $g = 2$ for charged elementary particles with spin $\frac{1}{2}$
- Radiative corrections slightly increase g; the anomaly is defined as $a = \frac{g-2}{2}$
- QED prediction (first order, dominant) by Schwinger agreed with Kush and Foley results for electron anomaly (1948): $a = \frac{\alpha}{2\pi} \approx 0.00116$

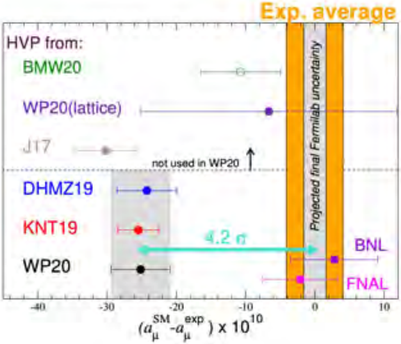




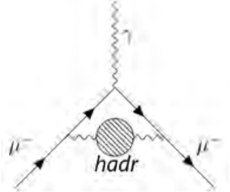
The $g-2$ theoretical and experimental values

- Run-1 results from FNAL (April '21) bring the combined experimental uncertainty down to 0.35 ppm
- Discrepancy with theoretical value recommended by White Paper 2020 [T. Aoyama et al.]:

$$a_\mu^{exp} - a_\mu^{SM} = 251(59) \times 10^{-11}$$



- Target uncertainty at FNAL is 0.14 ppm
- Major systematic uncertainty on theoretical value comes from HVP contribution ($\delta a_\mu^{HVP} \sim 4 \cdot 10^{-10}$)



Sections

1. Measurement principle in the $g-2$ experiment at Fermilab
2. Muon g-2 at Fermilab
3. Precession Frequency Analysis: fits and systematic studies
4. Conclusions

Measurement principle in the $g-2$ experiment at Fermilab

Spin Precession in a Magnetic Field

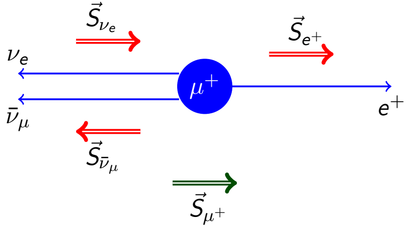
$g > 2$: anomalous precession of spin in B-field, defined as $\vec{\omega}_a = \vec{\omega}_{spin} - \vec{\omega}_{cyclotron}$:

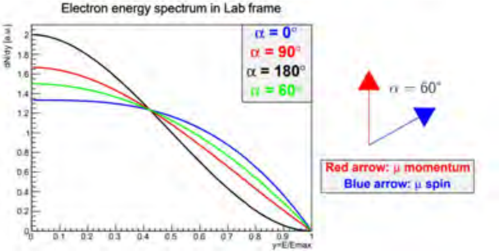
$$\vec{\omega}_a = -\frac{e}{mc}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E} - a_\mu \frac{\gamma}{\gamma+1}\left(\vec{\beta}\cdot\vec{B}\right)\vec{\beta}\right]$$

- The electric field term cancels out with choice of $\gamma \sim 29.3$ ($p_\mu \sim 3.094\,\mathrm{GeV/c}$ is the "muon magic momentum")
- The magnetic field term cancels out since beam trajectory is perpendicular to B-field

The spin precession period in the Muon g-2 experiment is $4.4\,\mathrm{\mu s}$: the spin spans $\sim 12^\circ$ per-turn ($149.2\,\mathrm{ns}$)

Parity violation in muon decay



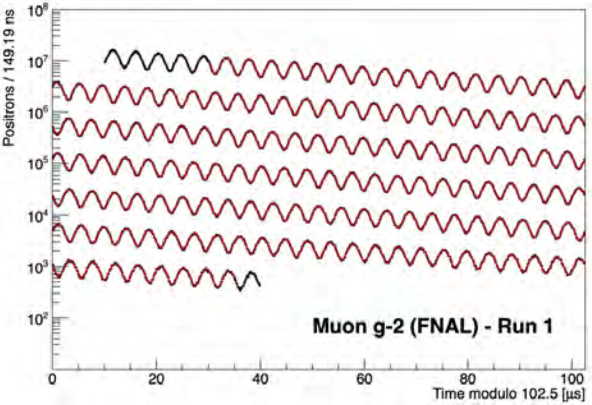


- In the muon rest frame, high energy positrons are emitted preferentially in the muon's spin direction
- In the lab frame, the energy spectrum of decayed positrons changes according to the anomalous precession phase
- The number of positrons above a certain energy threshold oscillates in time with the ω_a precession frequency

Wiggle Plot: count e^+ above threshold over time

The anomalous precession frequency is extracted from a fit to the so-called "wiggle plot". $\gamma\tau$ is the muon lifetime in the lab frame $\sim 64.4\,\mu\mathrm{s}$.

$$N(t) = N_0 e^{-t/\gamma\tau}\left[1 + A \cdot \cos(\omega_a \cdot t + \varphi)\right]$$



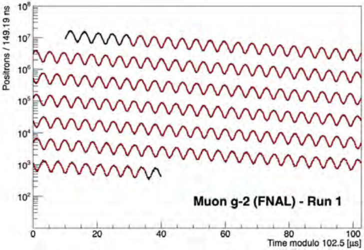
From ω_a to a_μ

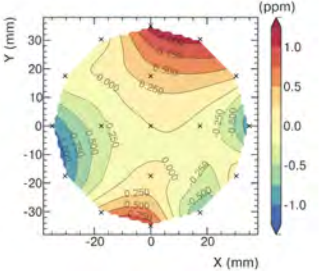
The master formula (simplified version) for a_μ is:

$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T)} \underbrace{\frac{\mu'_p(T)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}}_{\text{External}}$$

where the B-field is expressed in term of the shielded proton precession frequency, measured by NMR probes (see R. Reimann's talk on July 7th). We extract the ratio:

$$R'_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T)}$$





Muon g-2 at Fermilab

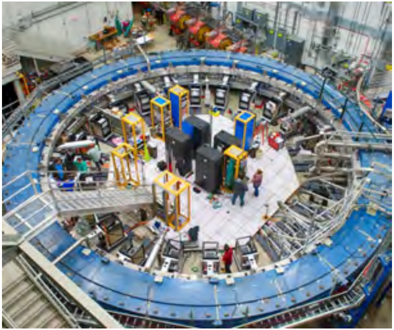
Experiment at Fermilab Muon Campus

Ph: Ryan Postel

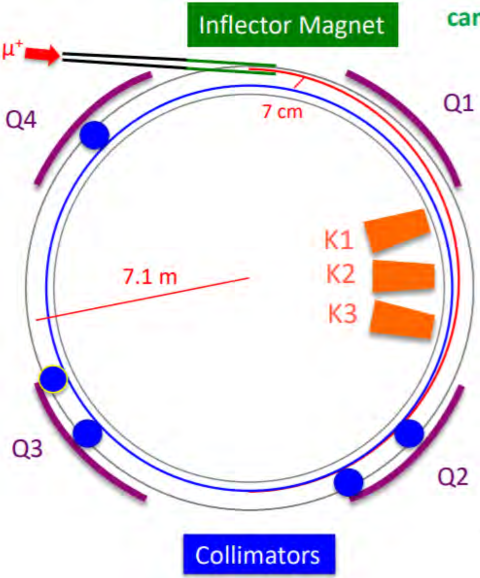
Ph: Reidar Hahn

Accelerator complex and storage ring

- $8\,\mathrm{GeV}$ protons collide on Inconel target, producing pions
- Pions decay into muons along $\sim 2\,\mathrm{km}$ line
- Muons are injected into the storage ring, in $1.45\,\mathrm{T}$ B-field



The g-2 storage ring



cancels 1.45 T main field

$R_{\text{fill}} \sim 13\,\text{Hz}$

$N_\mu/\text{fill} \sim 10^4$

$N_{e^+\,E>1.8\,\text{GeV}}/\text{fill} \sim 5 \times 10^2$

Kicker magnets x3 (40-55 kV)

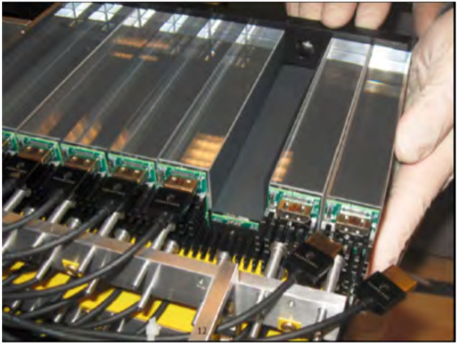
radially center beam

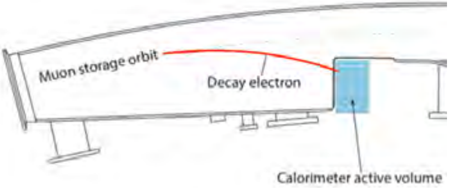
Focussing quadrupoles 15-28 kV

vertically focus beam

Calorimeter to detect decay positrons

- 24 calorimeters along the inner circumference (out of vacuum)
- 54 PbF_2 crystals ($n = 1.82$) in a 6×9 matrix
- Each crystal is $2.5 \times 2.5 \times 14\ \text{cm}^3 \sim 15\ X_0$ for PbF_2
- Čerenkov light is read by large area SiPM
- Gain is monitored at a 10^{-4} level of stability by state of the art Laser Calibration System

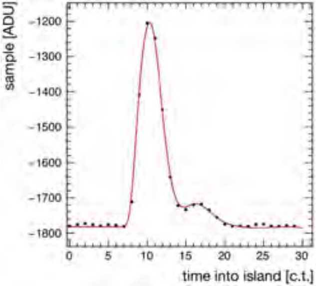




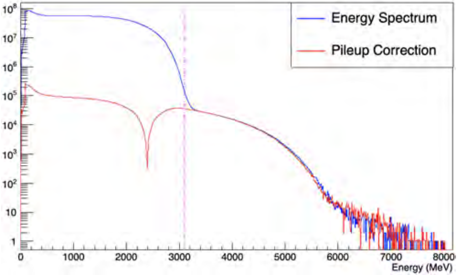
Precession Frequency Analysis: fits and systematic studies

ω_a analysis: particles reconstruction

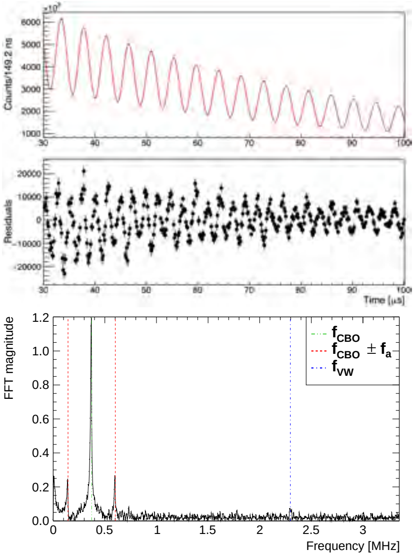
- Positron hit on the calorimeter crystals: raw data is fitted to identify pulses
- Clustering algorithms to reconstruct time and energy of crystal hits
- Pileup subtraction to correct for double and triple overlapping positrons, which show up in energy spectra



- Run1 result was a combination of 4 different analyses, each with its own reconstruction chain. Run-2/3: 7 ω_a analysis teams.



Fitting ω_a

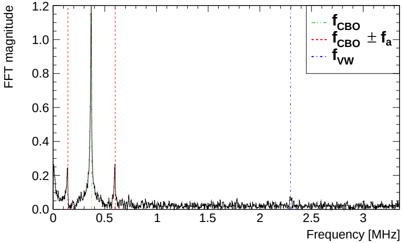


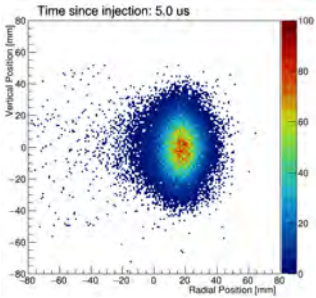
In principle simple fit:

$$N(t) = N_0 e^{-t/\tau_\mu}\left[1 + A\cos(\omega_a t + \varphi)\right]$$

Beam dynamic effects appear in the residuals FFT

Fitting ω_a





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- Beam oscillations (CBO): beam profile changes over time, muons oscillate coherently with $T \approx 2.7\,\mu\mathrm{s}$

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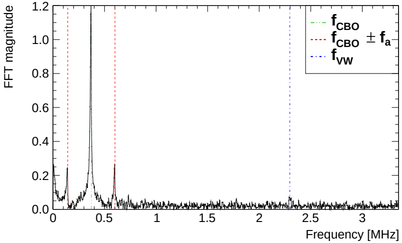

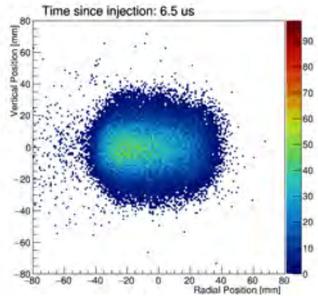

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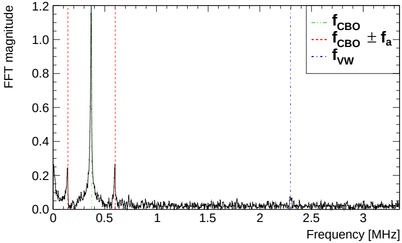

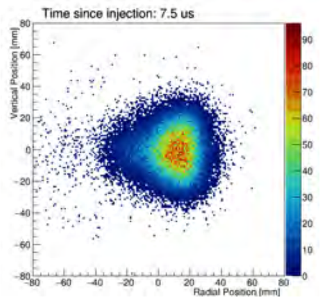

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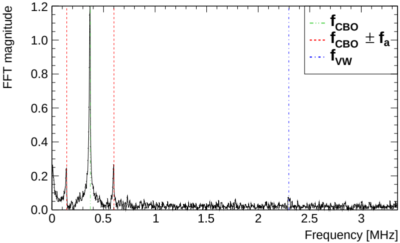

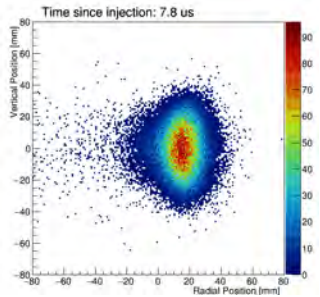

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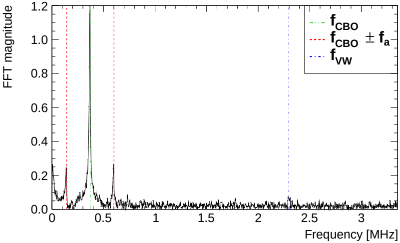

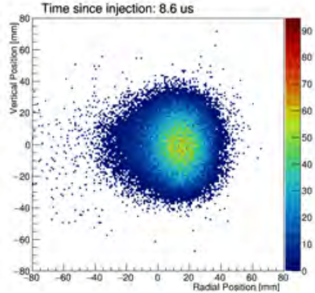

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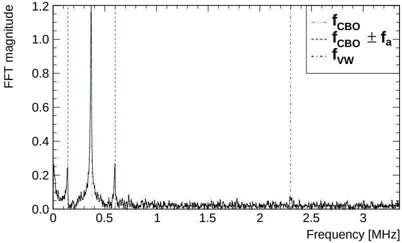

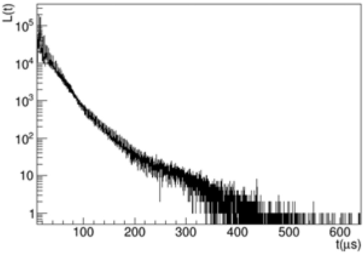

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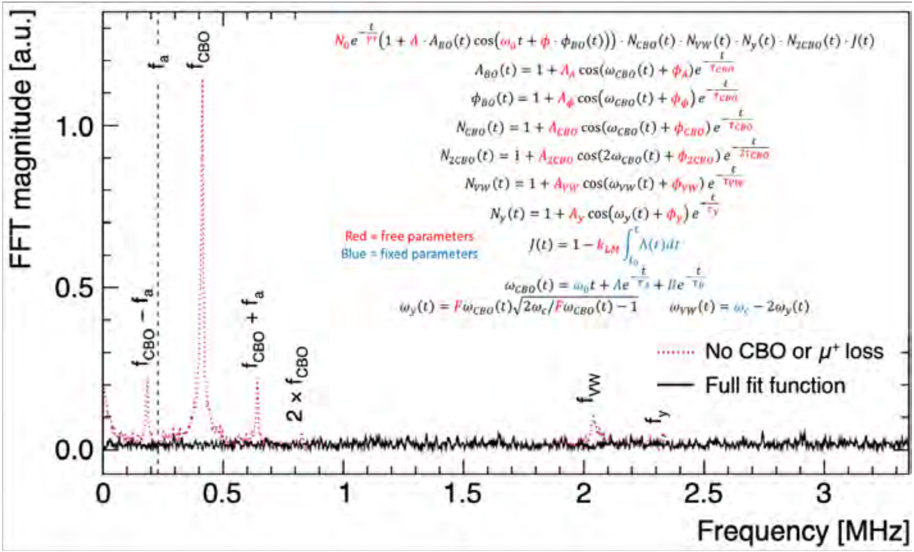
Beam dynamic effects appear in the residuals FFT:

- Beam oscillations (CBO): beam profile changes over time, muons oscillate coherently with $T \approx 2.7\,\mu\mathrm{s}$
- Muon losses: a fraction of muons drifts out of the storage ring over time and hit multiple calorimeters

Residuals FFT



$$N_0 e^{-\frac{t}{\gamma\tau}}\left(1 + A \cdot A_{BO}(t)\cos\left(\omega_a t + \phi \cdot \phi_{BO}(t)\right)\right) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_y(t) \cdot N_{2CBO}(t) \cdot J(t)$$

$$A_{BO}(t) = 1 + A_A \cos(\omega_{CBO}(t) + \phi_A) e^{-\frac{t}{\tau_{CBO}}}$$

$$\phi_{BO}(t) = 1 + A_\phi \cos(\omega_{CBO}(t) + \phi_\phi) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{CBO}(t) = 1 + A_{CBO} \cos(\omega_{CBO}(t) + \phi_{CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{2CBO}(t) = 1 + A_{2CBO} \cos(2\omega_{CBO}(t) + \phi_{2CBO}) e^{-\frac{t}{2\tau_{CBO}}}$$

$$N_{VW}(t) = 1 + A_{VW} \cos(\omega_{VW}(t) + \phi_{VW}) e^{-\frac{t}{\tau_{VW}}}$$

$$N_y(t) = 1 + A_y \cos(\omega_y(t) + \phi_y) e^{-\frac{t}{\tau_y}}$$

Red = free parameters
Blue = fixed parameters

$$J(t) = 1 - k_{LM} \int_{t_0}^{t} \Lambda(t) dt$$

$$\omega_{CBO}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

$$\omega_y(t) = F\omega_{CBO}(t)\sqrt{2\omega_c / F\omega_{CBO}(t) - 1} \qquad \omega_{VW}(t) = \omega_c - 2\omega_y(t)$$

Detailed ω_a systematics and projections for Run-2/3

Run-1 main ω_a systematics

	Value [ppb]
Uncertainty (stat.)	434
Uncertainty (syst.)	56
Detailed Systematics	
Time Randomization	9
Time Correction	1
Gain	8
Pileup	35
Pileup Artificial Dead Time	3
Muon Loss	3
CBO (beam oscillations)	38
Residual Slow Term	17

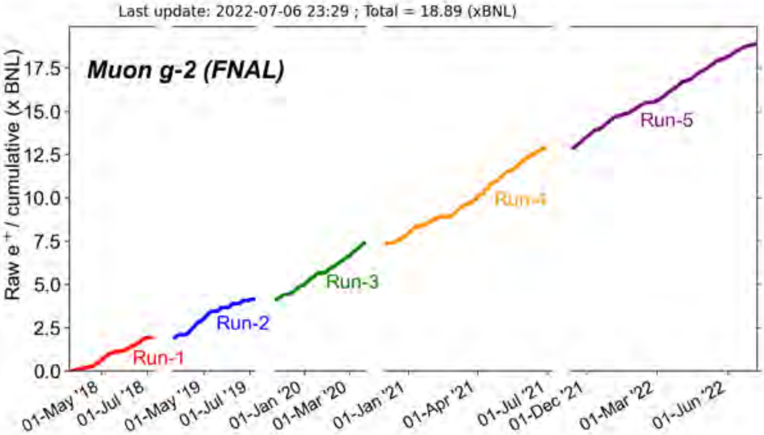
Hardware and analysis improvements:

- Stabilized temperature in the hall and in the magnet
- Stronger kick to better center the beam
- New reconstruction to better resolve pileup events

The major systematics in Run-1 will be reduced by a factor ~ 2.

Run-2/3 statistics

In 2019 and 2020 we collected ~ 4.5 more statistics than 2018: statistical uncertainty will decrease from 434 ppb to O(200 ppb) in Run-2/3.



Conclusions

Summary and conclusions

- The Muon g-2 experiment at Fermilab measures a fundamental property of the muon at very high precision
- The anomalous precession frequency and the magnetic field in the ring are measured in order to obtain the anomaly a_μ
- First result in 2021 was in very good agreement with the previous BNL measurement: 4.2 sigma discrepancy between theoretical (recommended value from the Theory Initiative) and combined experimental value
- $x4$ times statistics and upgrades to the machine and the analysis technique in Run-2/3 improved the systematic uncertainties on ω_a by a factor of ~ 2

Muon g-2 Collaboration

Thank you for your attention!

Any questions?

4 papers for Run1 result, 2021

Collaboration meeting at Elba, 2019

BACKUP SLIDES

References

- B. Abi et al. (Muon $g-2$ Collaboration) - *Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm*, Phys. Rev. Lett. 126, 141801
- T. Albahri et al. (Muon $g-2$ Collaboration) - *Measurement of the anomalous precession frequency of the muon in the Fermilab Muon $g-2$ Experiment*, Phys. Rev. D 103, 072002
- T. Albahri et al. (Muon $g-2$ Collaboration) - *Magnetic-field measurement and analysis for the Muon $g-2$ Experiment at Fermilab*, Phys. Rev. A 103, 042208
- T. Albahri et al. (Muon $g-2$ Collaboration) - *Beam dynamics corrections to the Run-1 measurement of the muon anomalous magnetic moment at Fermilab*, Phys.Rev.Accel.Beams 24 (2021) 4, 044002
- T. Aoyama et al. - *The anomalous magnetic moment of the muon in the Standard Model*, Phys. Rept. 887 (2020), 1-166

A new experiment at FNAL

The Muon g-2 (E989) at Fermilab aims to reduce the uncertainty on the anomalous magnetic moment by a factor 4 (540 ppb $\rightarrow$ 140 ppb):

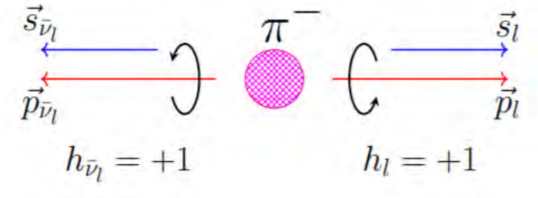
- Fermilab's accelerator to produce μ^+ beam: higher rate, more clean beam (proton separation, 2 km decay tunnel for π^+)
- 21 times BNL data to reduce statistical uncertainty
- Better beam storage and tracking
- More uniform magnetic field
- Improved calorimeters: segmented Čerenkov crystals (to reduce pileup)
- Laser calibration system to correct for gain changes

TDR: projections of syst. uncert. on ω_a

Source	E821 [ppb]	E989 improvements	E989 goal [ppb]
Gain correction	120	Better laser calibration Low-energy threshold	20
Pileup	80	Low-energy samples recorded Calorimeter segmentation	40
Lost muons	90	Better collimation in ring	20
CBO	70	Higher CBO frequency Better match of beamline to ring	< 30
E-field and pitch	50	2 tracker stations Precise storage ring simulations	30
Total	180		70

Polarized muons: parity violation in pion decay

Muons emitted along flight direction are polarized.



From ω_a to a_μ (for real)

The final "master" formula for R'_μ, with all the corrections is:

$$R'_\mu = \frac{\omega_a}{\tilde{\omega}'_p(T)} = \frac{f_{clock} \cdot \omega_a^{meas} \cdot \overbrace{(1 + C_e + C_p + C_{ml} + C_{pa})}^{\text{Beam Dynamics}}}{f_{calib} \cdot \left\langle \omega'_p(x,y,\phi) \cdot M(x,y,\phi) \right\rangle \cdot \underbrace{(1 + B_k + B_q)}_{\text{Transient Fields}}}$$

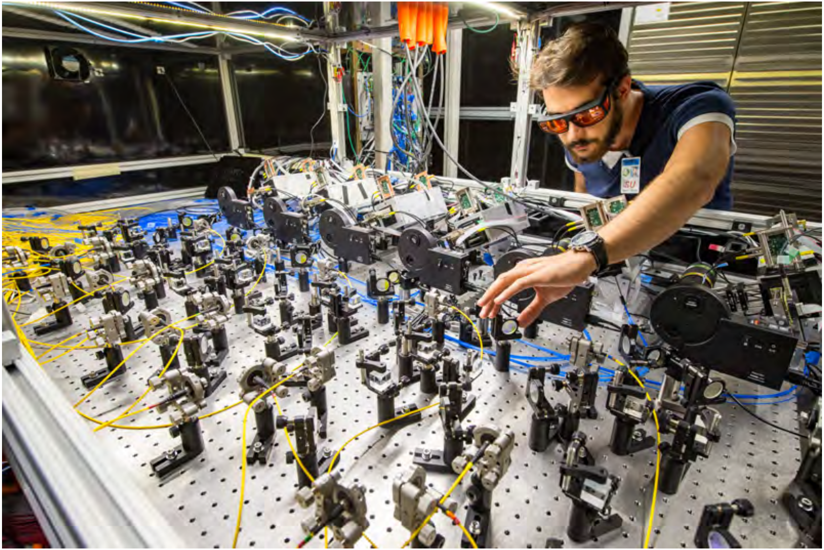
- ω_a^{meas} is the the measured precession frequency (this talk)
- $\tilde{\omega}'_p(T)$ is the the magnetic field magnitude (in term of NMR frequency) around the ring (R. Reimann's talk)
- $C_e + C_p + C_{ml} + C_{pa}$ are beam dynamics corrections (A. Driutti's talk)
- f_{clock} is the main clock frequency (blinded)

A blind analysis

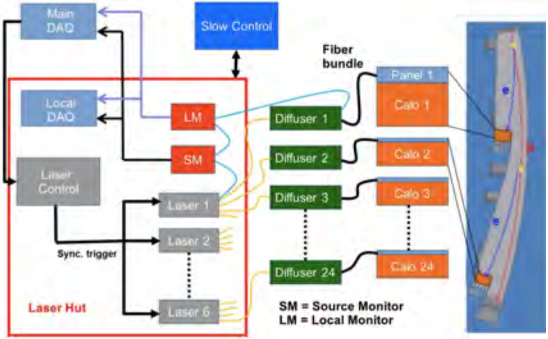
To avoid biases from knowing the a_μ from BNL a two-level blinding to the ω_a analysis has been applied:

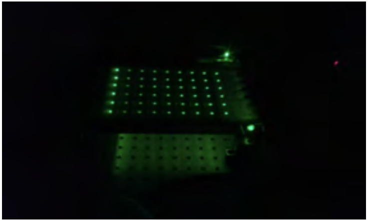
- Hardware: the main clock is tuned at $(40-\varepsilon)$ MHz; ε is unknown to all the collaboration
- Software: each analyzer uses a different offset in the ω_a fit

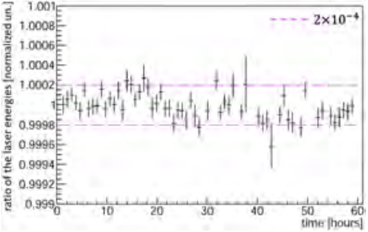
Laser Calibration System



Laser Calibration System



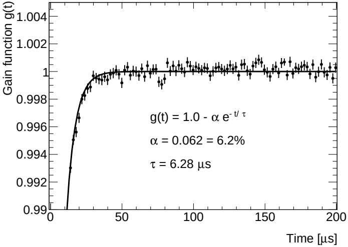




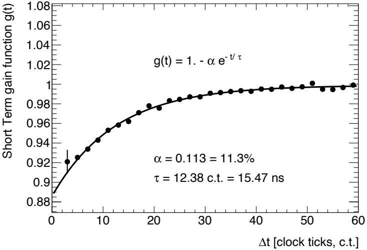
Calibration of the 1296 crystals for Short Term $\mathcal{O}(1\text{ ns})$, Measurement Window $\mathcal{O}(1\text{ ms})$ and Long Term $\mathcal{O}(1\text{ day})$

Gain corrections

Laser based gain monitoring of the SiPMs

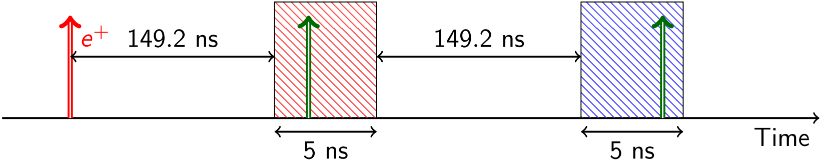


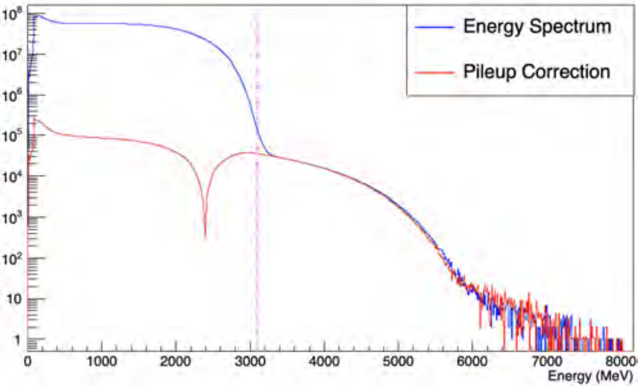
Gain function to correct for the SiPM bias voltage recovery time due to the large number of particles hitting the calorimeters. Laser pulses (known energy) are sent in the calorimeters at different times to measure the response function.



Gain function to correct for the SiPM recovery when two positrons hit the crystals within a short period of time (< 100 ns). It depends on the positrons energy and it's measured in dedicated laser runs.

Pileup subtraction





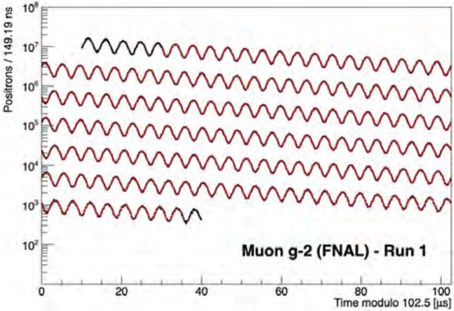
ω_a analysis: different methods

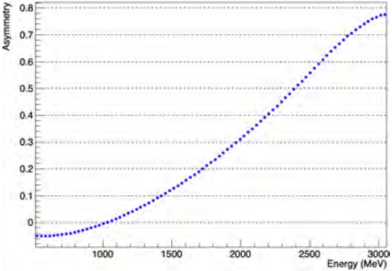
T: integrate all positrons above 1.7 GeV

A: weight the positrons with asymmetry function and integrate above 1.1 GeV

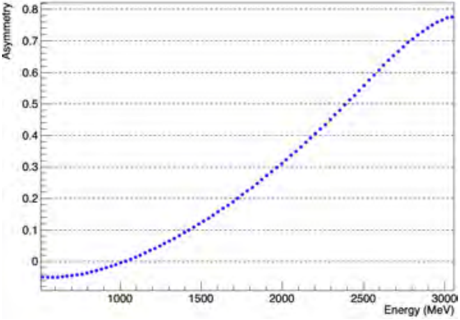
R: randomly split dataset in 2 subsets shifted by ±half a g-2 period and combined remove slow terms (exponential, gain…). Also R-A weighted.

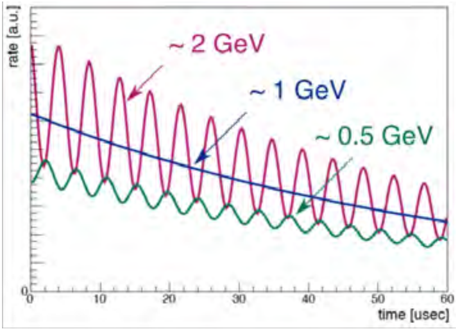
Q: No clustering: just integrate energy above threshold for each crystal





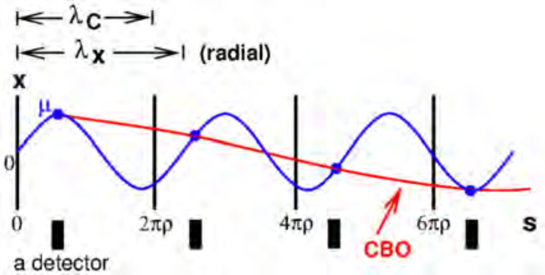
Asymmetry Function





$$N(t) = N_0 e^{-t/\tau} \left[1 + A(E) \cdot \cos(\omega_a \cdot t + \varphi)\right]$$

Coherent Betatron Oscillation: CBO

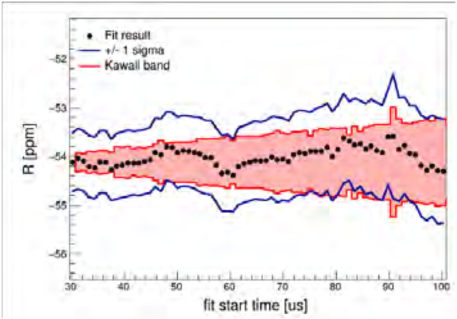


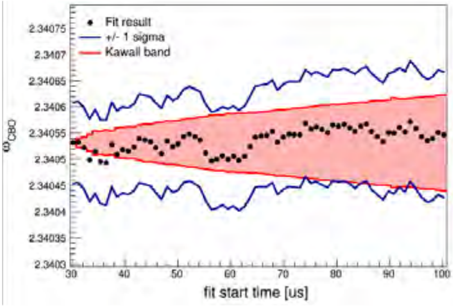
Radial and vertical beam oscillation: coherent effect in calorimeter wiggle plots.

Cyclotron frequency: $f_C \approx 6.7\,\text{MHz}$ (period of $0.149\,\mu\text{s}$).
Radial coherent oscillation: $f_{CBO} \approx 0.37\,\text{MHz}$ (period of $2.68\,\mu\text{s}$).
Vertical coherent oscillation: $f_{VW} \approx 2.3\,\text{MHz}$ (period of $0.46\,\mu\text{s}$).

Start Time Scans

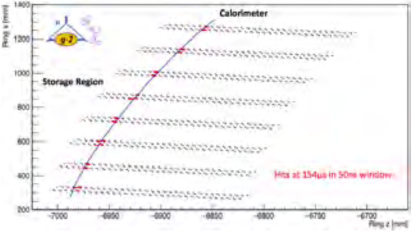
Among the many consistency checks of the fit there are the start time scans: the fit start time is varied in order to check its stability

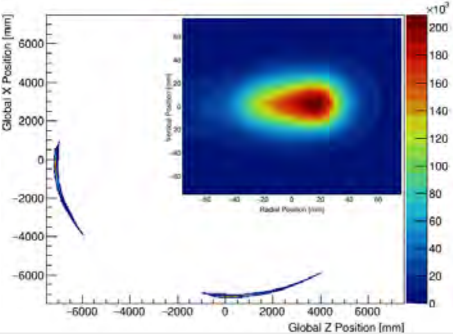




Tracker Detector

- 2 tracker stations at 180° and 270°
- Each tracker has 8 modules
- Each module has 32 straw tubes in stereo pattern (give x and y)
- Reconstruct the position of the beam during the run
- Monitor the beam motion (more later)



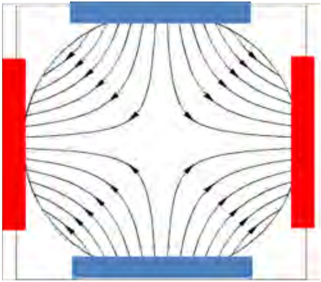


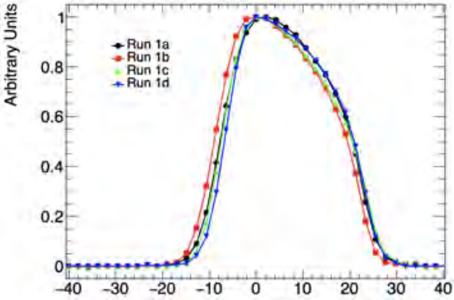
Electric field correction

Electric field correction due to the quadrupole field (seen as B-field in the muon rest frame).

$$\vec{\omega}_a = -\frac{e}{mc}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E}\right]$$

$$C_e = 2n(1-n)\beta^2\frac{\langle x_e^2\rangle}{R_0}$$



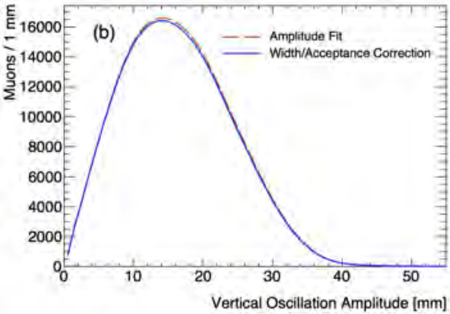


Pitch correction

Pitch correction due to the vertical oscillation of the muon beam

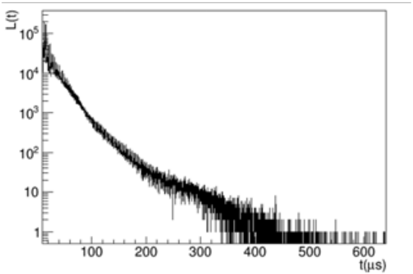
$$\vec{\omega}_a = -\frac{e}{mc}\left[a_\mu \vec{B} - a_\mu \frac{\gamma}{\gamma+1}\left(\vec{\beta}\cdot\vec{B}\right)\vec{\beta}\right]$$

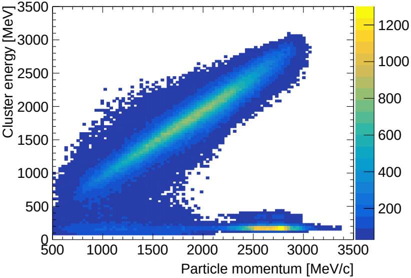
$$C_p = \frac{n}{4}\frac{\langle A^2 \rangle}{R_0^2}$$



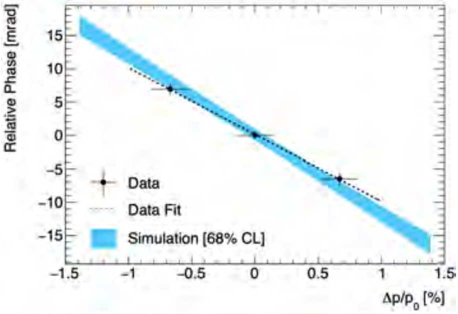
Lost Muons

- Muons outside the phase space hit the collimators and exit the storage region during the fill
- Can be reconstructed searching for:
  - MIP signals ($E \sim 170$ MeV) in the calorimeters
  - Coincidences between calorimeters with timing $\Delta t = 6.25$ ns
  - Tracker identification using the energy-momentum relation

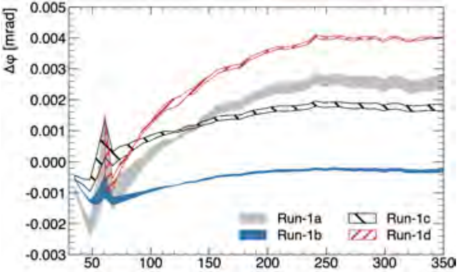




Muon Loss Correction



Muon momentum-phase correlation and loss rate depends on momentum lead to tiny phase shift



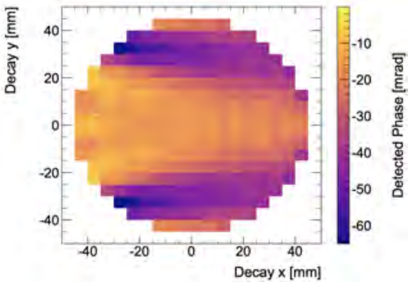
Phase-Acceptance Correction

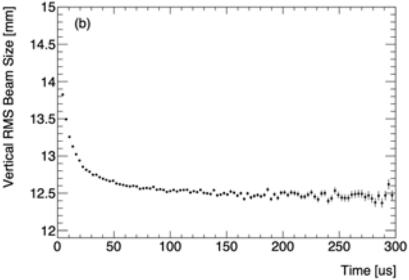
Measured phase depends on the decay point

Systematic shift of the beam early-to-late

Evaluated using beam simulation and tracker data

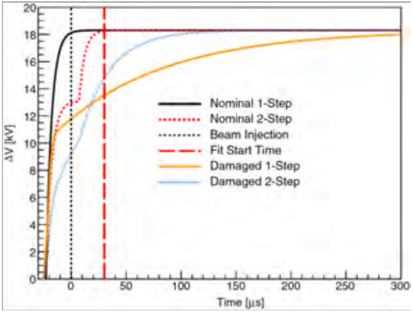
Fixed resistors in Run2 should improve this correction and systematic

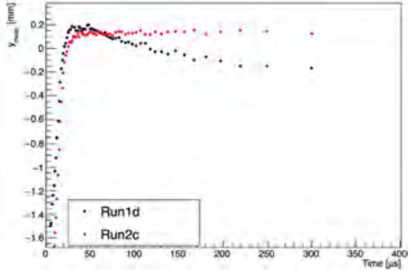




ESQ Resistors

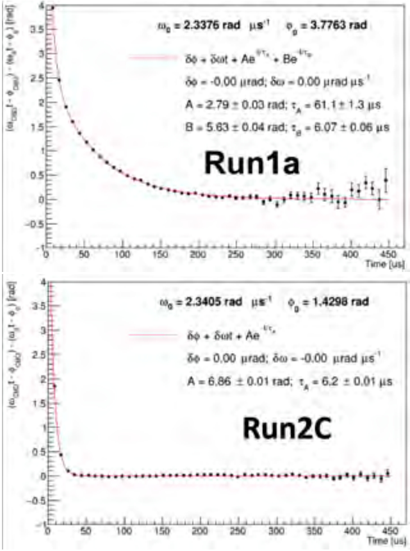
- Faulty resistors in one quadrupole: slower charge lead to unexpected beam vertical motion
- Beam moving down and increasing its RMS during the fill –> Stable in Run-2/3 due to repair of faulty resistors
- These variations in the beam induced a variable phase systematic $C_{pa} \sim \mathcal{O}(100 \text{ ppb})$ in the ω_a measure –> Reduced by a factor 2 in the Run2+3 analysis



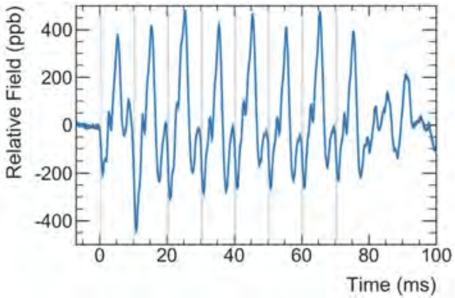


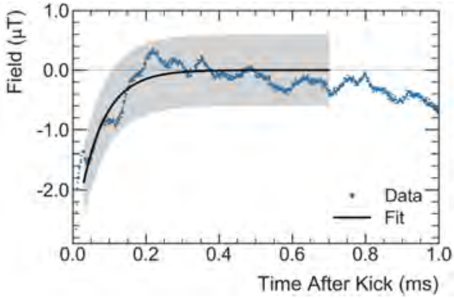
ESQ Resistors

- In Run-1:
  - The change in the quadrupole voltage induced a change in the beam frequencies
  - Added a variable CBO term in the fitting equation in order to account for the beam motion change
- In Run 2:
  - No more slow variation terms observed in the ω_a fit
  - Expected variation in the CBO frequency due to scraping



Transient fields





Transient B-field from the quadrupole plates vibration during the pulse. The grey bands represent the measurement window.

Transient B-field ∼ 200 G from the kicker Eddy Currents. Measured with a *Faraday Effect* magnetometer.

The unblinding

After validation of all the analyses the whole collaboration met on the 25th of February 2021:

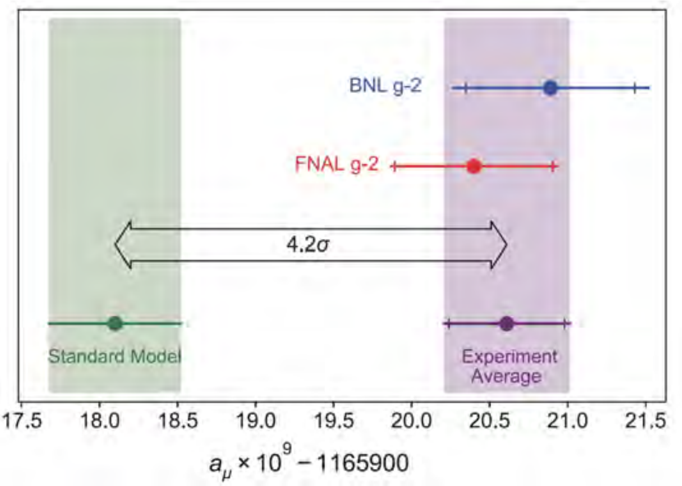
- Two envelopes with secret frequency were opened (one in Seattle (University of Washington, other in Fermilab)
- Unblinded result was computed

Final Result for Run-1

	Correction Factor [ppb]	Uncertainty [ppb]
ω_a (stat.)	–	434
ω_a (syst.)	–	56
f_b/f_0	–	2
C_e	489	53
C_p	180	13
C_{ml}	-11	5
C_{pa}	-158	75
$f_{calib} \left\langle \omega'_p(x,y,\phi) \cdot M(x,y,\phi) \right\rangle$	–	56
B_q	-17	92
B_k	-27	37
$\mu'_p(34.7^\circ C)/\mu_e$ [PCK77]	–	10
m_μ/m_e [LAMPF-99; CD-2018]	–	22
$g_e/2$ [HFG08]	–	0
Total Systematic	–	157
Total Fundamental Factors	–	25
Total	544	461

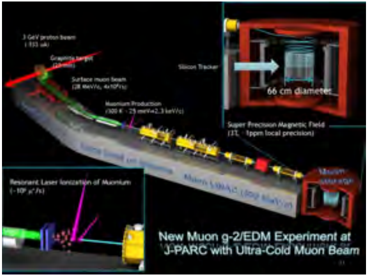
Muon g-2 First Result

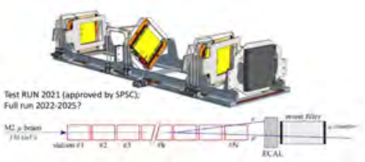
$$a_\mu^{FNAL} = 116\ 592\ 040(54) \times 10^{-11}\ [0.46\ \text{ppm}]$$



Other Experiments

- New independent method to measure g-2
- Re-accelerated muons at low energy
- Cross-check of BNL/FNAL





- Alternative measurement of hadronic loops (HVP) for the anomaly
- Scattering experiment with 150 GeV muons
- Cross-check of SM prediction
- See R. Pilato's talk