

Muon $g-2$ at Fermilab

René Reimann
on behalf of Muon $g-2$ Collaboration

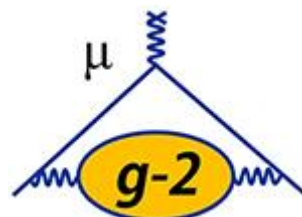
CLFV 2023, Heidelberg

June 21st, 2022



PRISMA+

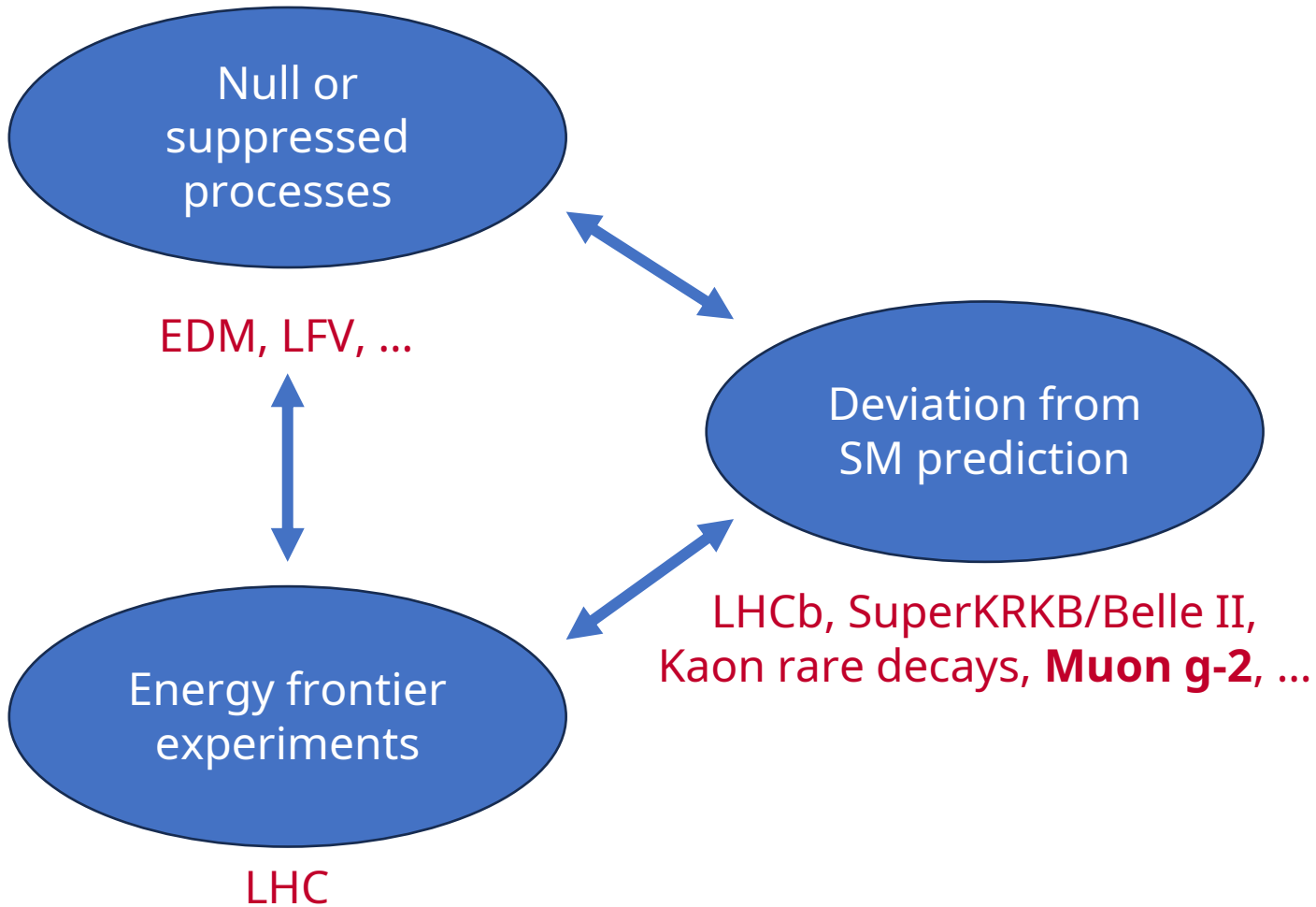
Cluster of Excellence
Precision Physics, Fundamental Interactions
and Structure of Matter



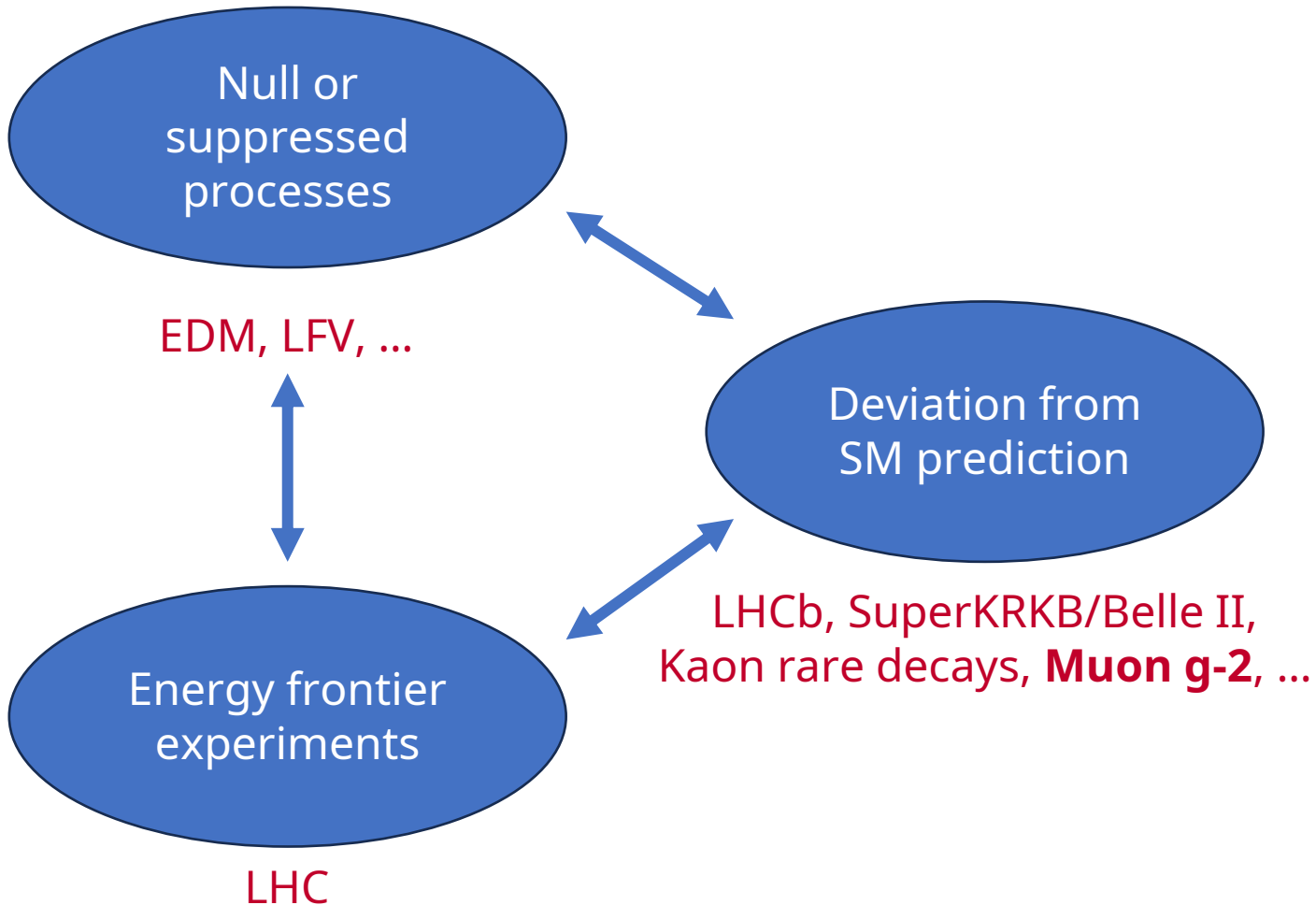
JOHANNES GUTENBERG
UNIVERSITÄT MAINZ



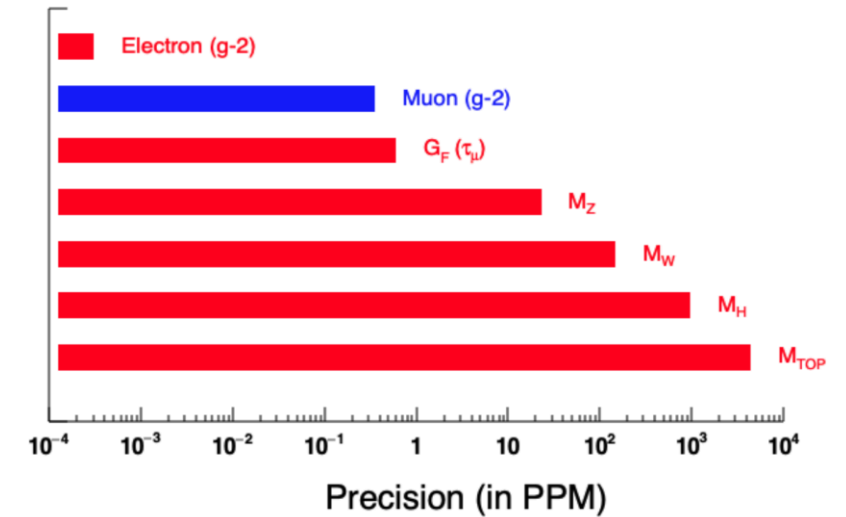
Approaches to new physics



Approaches to new physics



Most precise SM measurements



high precision measurement
+
high precision theory calculation
=
stringent SM test

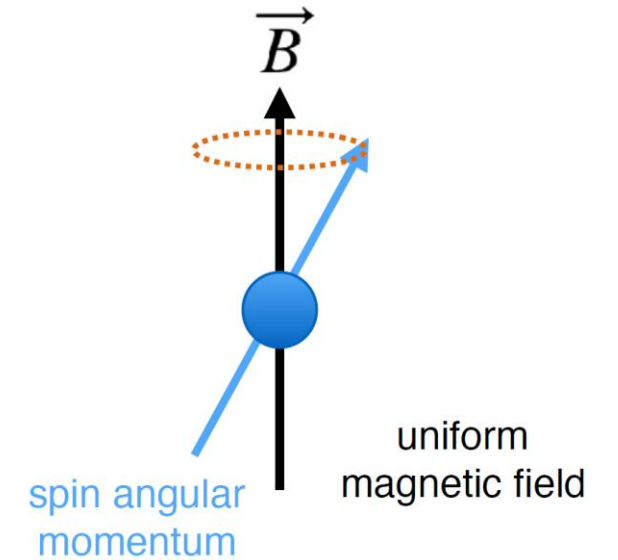
Muon magnetic moment

$$\vec{\mu}_\mu = -g_\mu \frac{e}{2m_\mu} \vec{S}$$

magnetic moment

proportionality constant

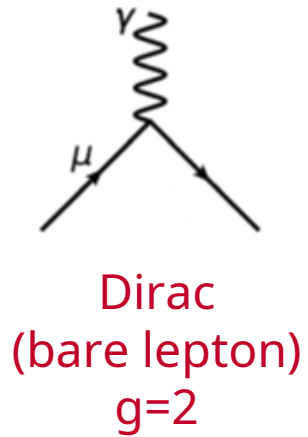
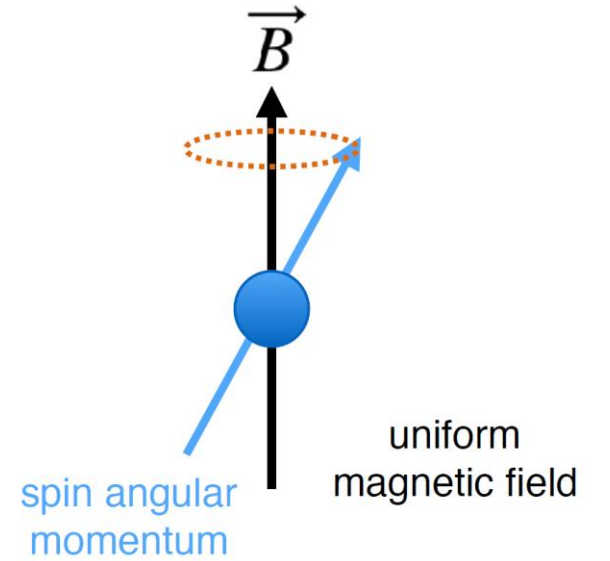
spin



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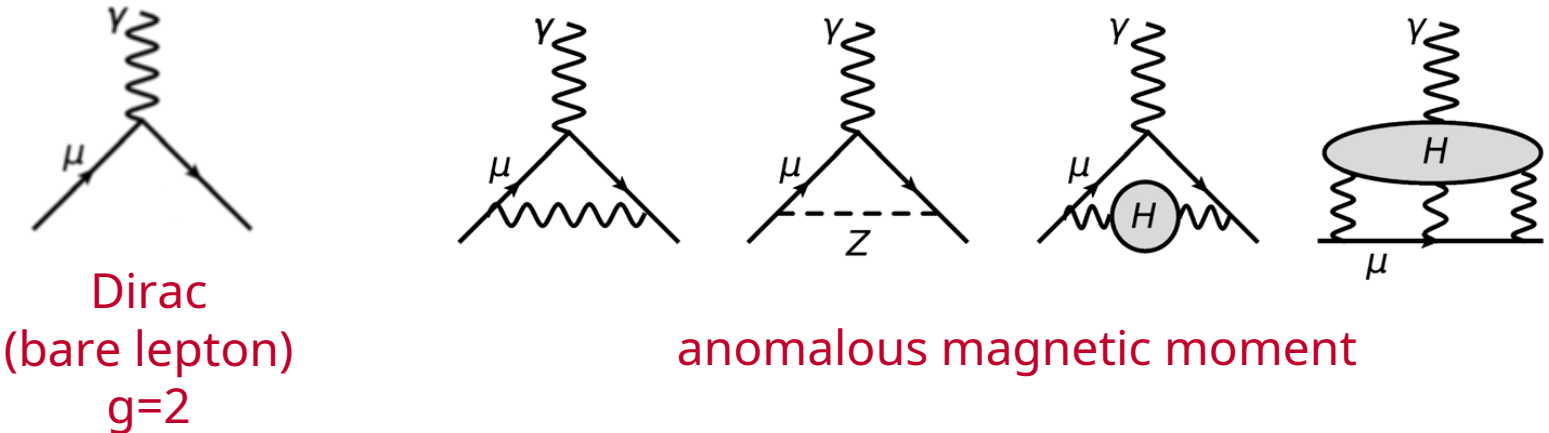
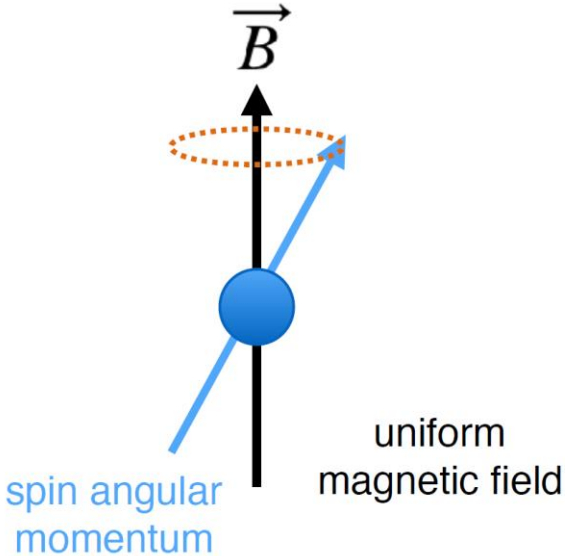
magnetic moment proportionality constant spin



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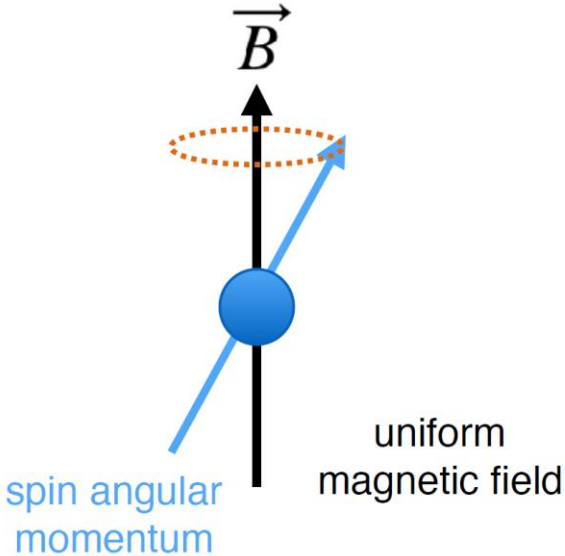
magnetic moment
↑
proportionality constant
spin



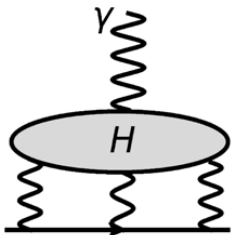
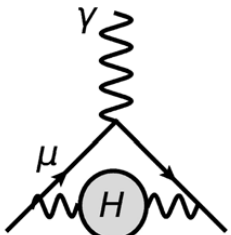
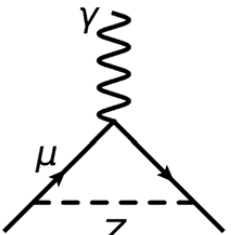
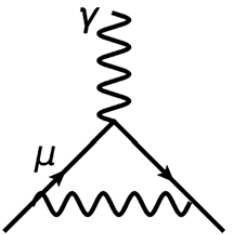
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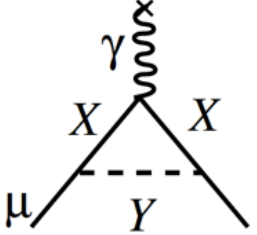
magnetic moment proportionality constant spin



Dirac
(bare lepton)
 $g=2$



anomalous magnetic moment

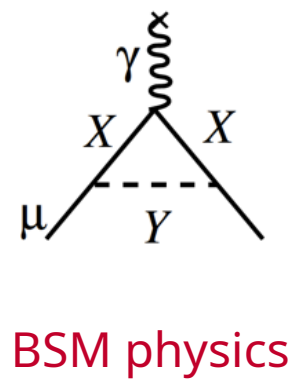
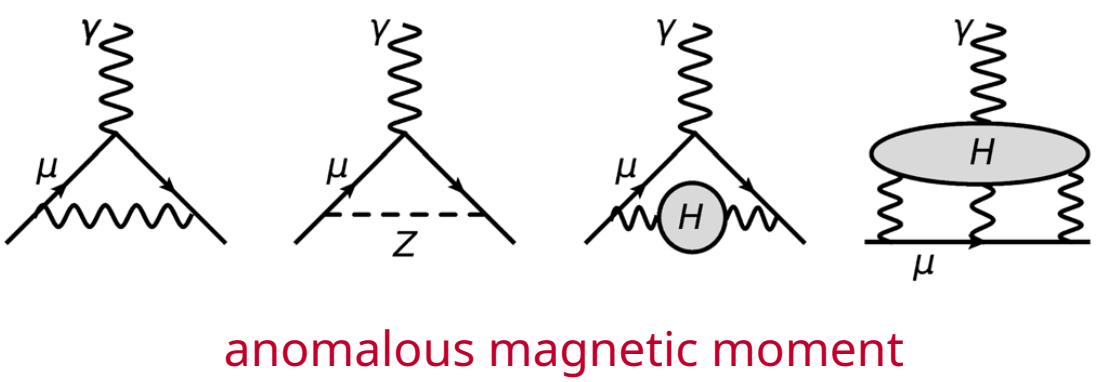
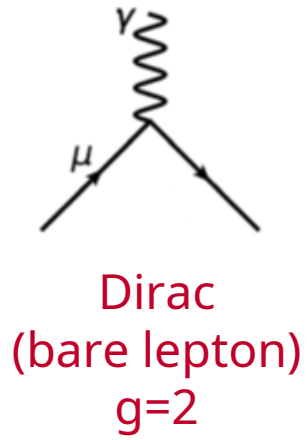
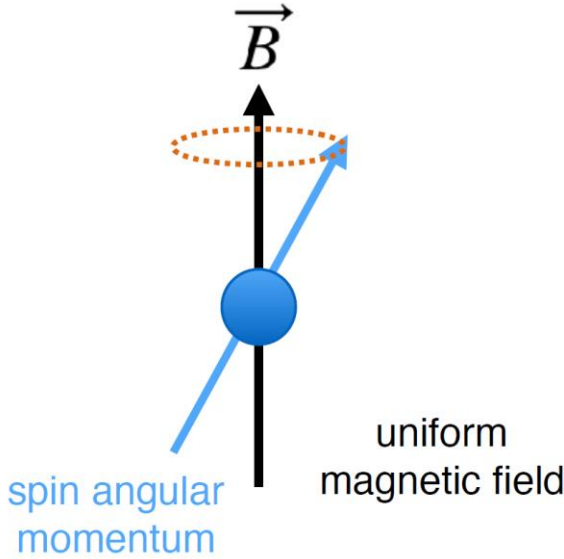


BSM physics

Muon magnetic moment

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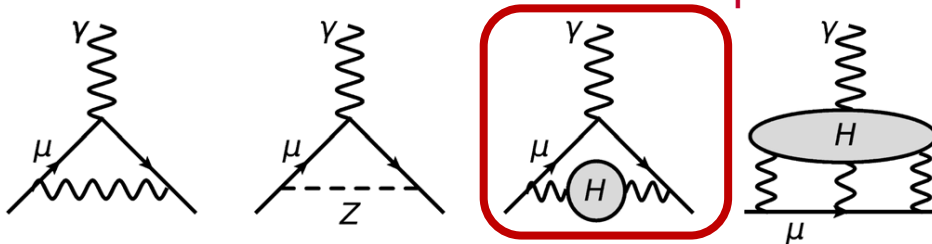
magnetic moment proportionality constant spin



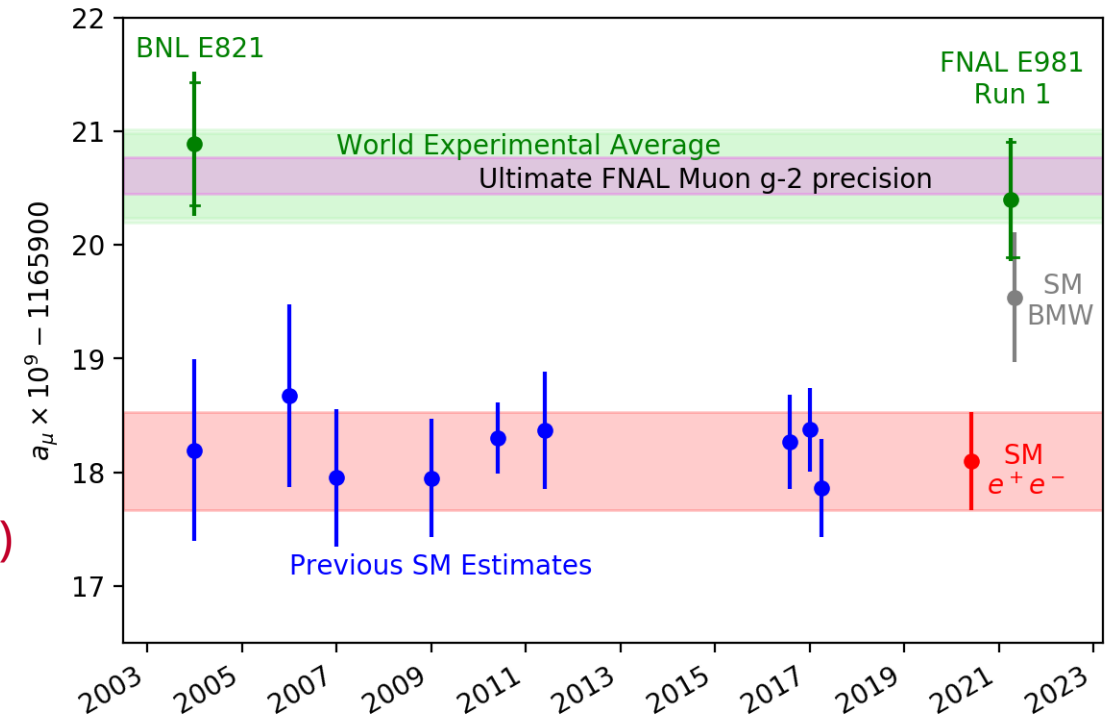
$$\Delta a_1^{\text{BSM}} \propto \frac{g_{\text{BSM}}}{16\pi^2} \frac{(\text{lepton mass})^2}{(\text{new particle mass})^2}$$

Current status

- Long standing discrepancy between theory calculation and experimental result
- Muon g-2 collaboration published Run 1 result
B. Abi *et al.* (Muon g-2 Collaboration) Phys. Rev. Lett. **126**, 141801, 2021
 - In agreement with BNL measurement
- Uncertainty in theory calculation dominated by calculation of hadronic vacuum polarization (HVP)

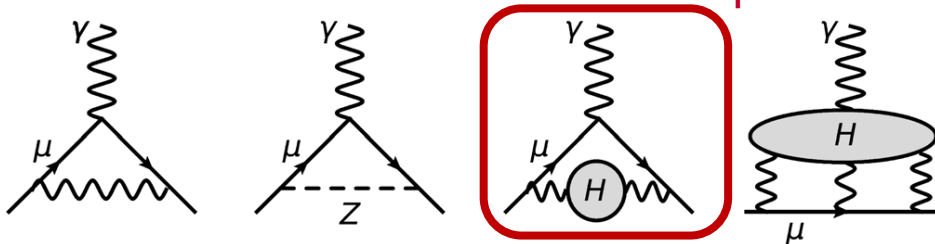


- Dispersive approach, 4.2σ tension
T. Aoyama *et al.*, Phys. Rept. **887** (2020) 1-166
- Lattice QCD approach, 1.5σ tension
Borsányi *et al.*, Nature **593**, 51-55, 2021 and arXiv:2002.12347



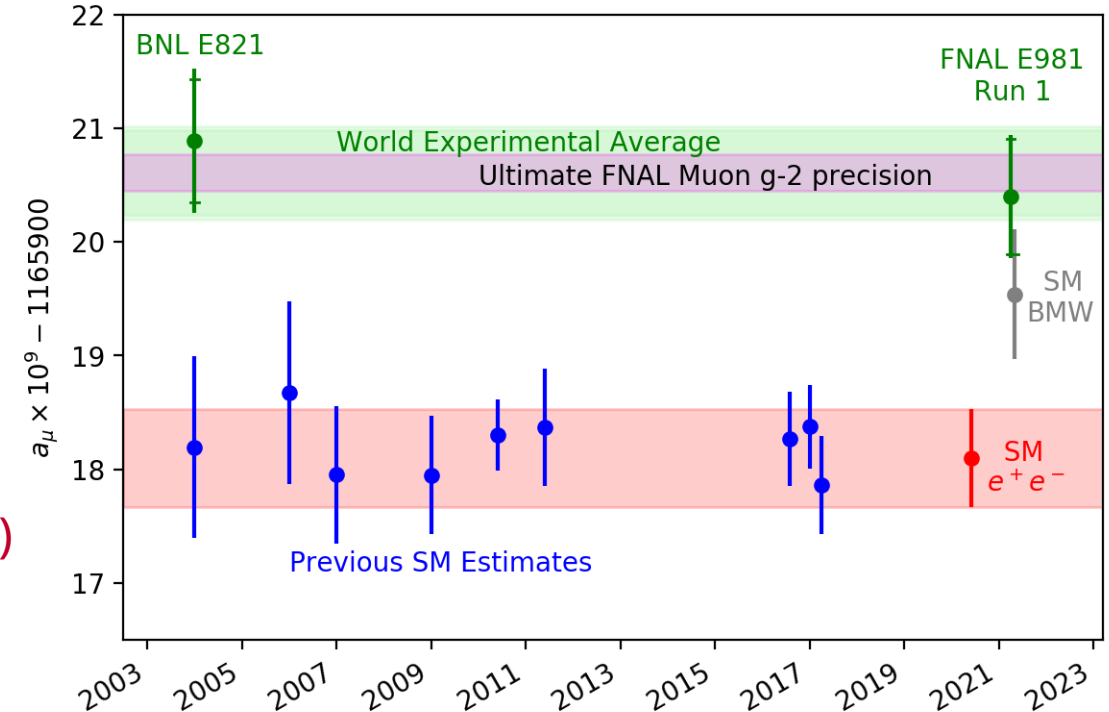
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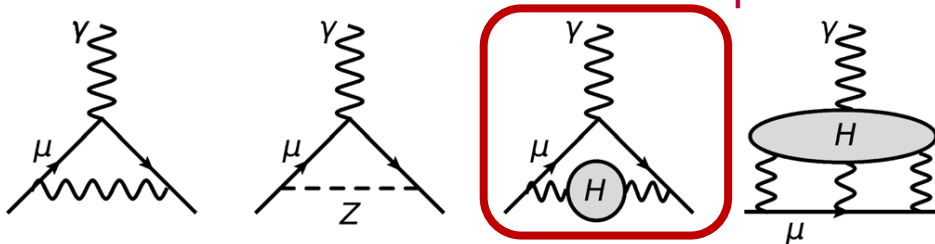
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See talk by
Ch. Lehner



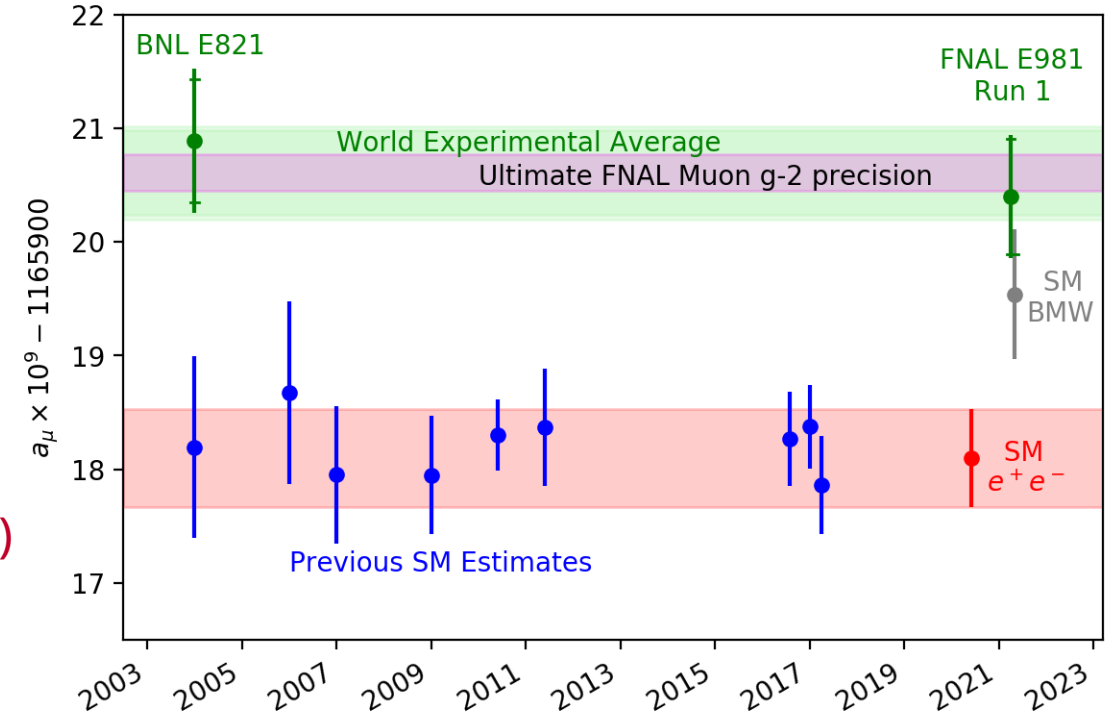
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See talk by Ch. Lehner



→ improve statistics & systematics of measurement

Muon in homogeneous magnetic field

Muon in homogeneous magnetic field

Cyclotron Motion

centrifugal force = Lorentz force

$$\vec{\omega}_C = -\frac{e}{m\gamma}\vec{B}$$

Muon in homogeneous magnetic field

Cyclotron Motion

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Spin Precession

magnetic moment and field couple

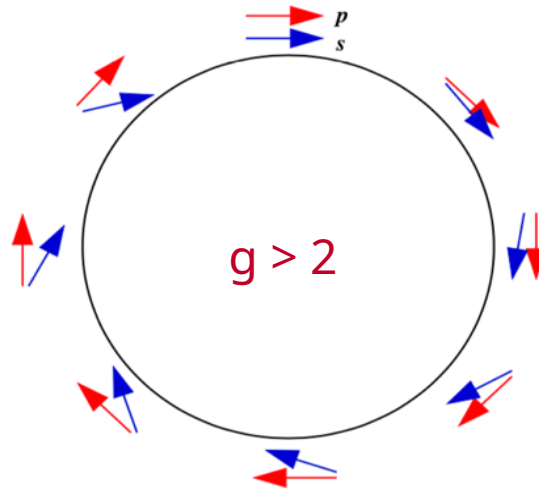
$$\vec{\omega}_S = -g\frac{e}{2m}\vec{B} - (1 - \gamma)\frac{e}{\gamma m}\vec{B}$$

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Spin Precession

magnetic moment and field couple

$$\vec{\omega}_S = -g\frac{e}{2m}\vec{B} - (1 - \gamma)\frac{e}{\gamma m}\vec{B}$$

$$\underbrace{\omega_S - \omega_C}_{\omega_a} = g_\mu \frac{e}{2m_\mu} B - \frac{e}{m} B = \underbrace{\frac{g_\mu - 2}{2}}_{a_\mu} \frac{e}{m_\mu} B$$

anomalous spin-precession
frequency

anomalous magnetic
moment

Relativistic muon in magnetic & electric fields

$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = \frac{e}{m} \left[a_\mu \vec{B} - a_\mu \left(\frac{\gamma}{\gamma + 1} \right) (\vec{\beta} \cdot \vec{B}) \vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

non-relativistic limit



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non-relativistic limit

electron motion
non-perpendicular
to magnetic field

cyclotron motion assumed motion
perpendicular to magnetic field

pitch of electron

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$$a_\mu^{SM} = 116591810(43) \times 10^{-11}$$

disappears for $\gamma \approx 29.3$

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magic momentum

$$p_\mu = 3.094 \text{ GeV}/c$$

Extracting a_μ

Anchor B , e and m_μ to other high-precision measurements and calculations

$$a_\mu = \frac{\omega_a m_\mu}{\tilde{B} e}$$

Extracting a_μ

Anchor B , e and m_μ to other high-precision measurements and calculations

$$a_\mu = \frac{\omega_a}{\tilde{B}} \frac{m_\mu}{e} = \frac{\omega_a}{\tilde{\omega}'_p(T_r)} \frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}$$
$$\tilde{B} = \frac{\hbar \tilde{\omega}'_p}{2\mu'_p}$$

proton spin-precession

Extracting a_μ

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$$\frac{\mu'_p(T_r)}{\mu_e(H)}$$

10.5 ppb uncertainty at $T_r = 34.7^\circ\text{C}$
Metrologia 13, 179 (1977)

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proton spin-precession

$$\frac{\mu_e(H)}{\mu_e}$$

Bound state QED calculation
exact
Rev. Mod. Phys. 88, 035009 (2016)

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Muonium hyperfine splitting
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Measurement
0.13 ppt uncertainty
Phys. Rev. Lett. 130, 071801 (2023)

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proton spin-precession

total uncertainty from
external quantities
25 ppb

$$\frac{\mu_e(H)}{\mu_e}$$

Bound state QED calculation
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We measure this ratio

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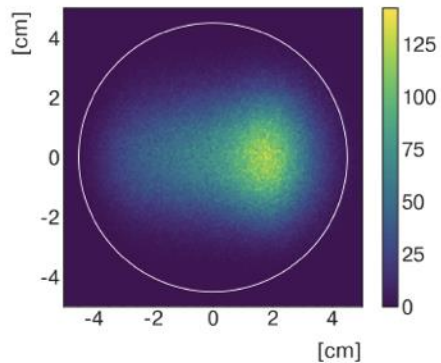
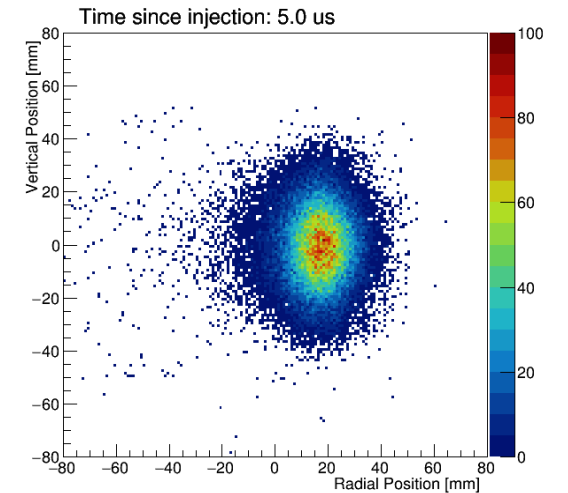
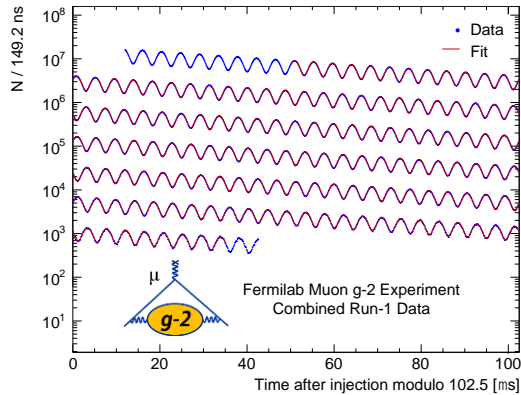
Extracting a_μ

Anomalous spin precession frequency

Muon beam dynamics corrections

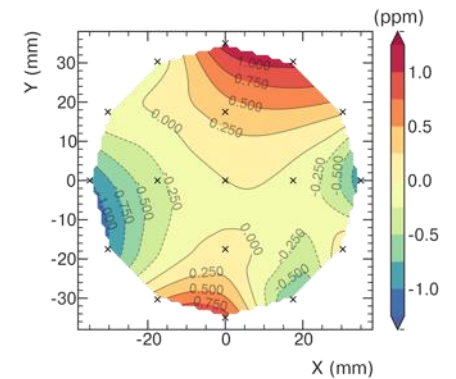
Clock blinding

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}} \omega_a^{\text{meas}} (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{\text{calib}} \langle M(x, y, \phi) \omega'_p(x, y, \phi) \rangle (1 + B_k + B_q)}$$

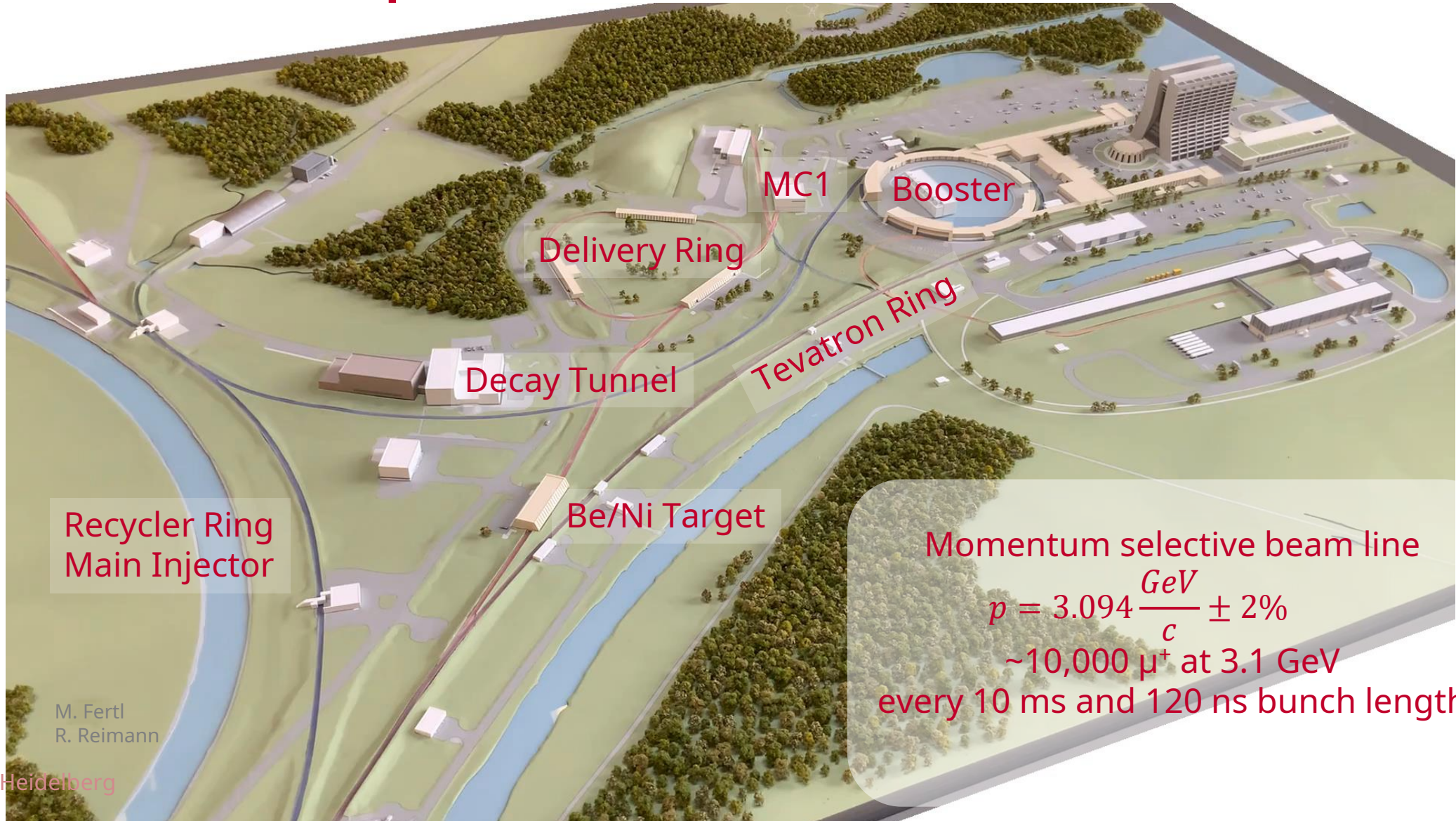


Spatial muon distribution

Magnetic field calibration
Spatial distribution of magnetic field
Transient magnetic fields



Muon Campus at Fermilab

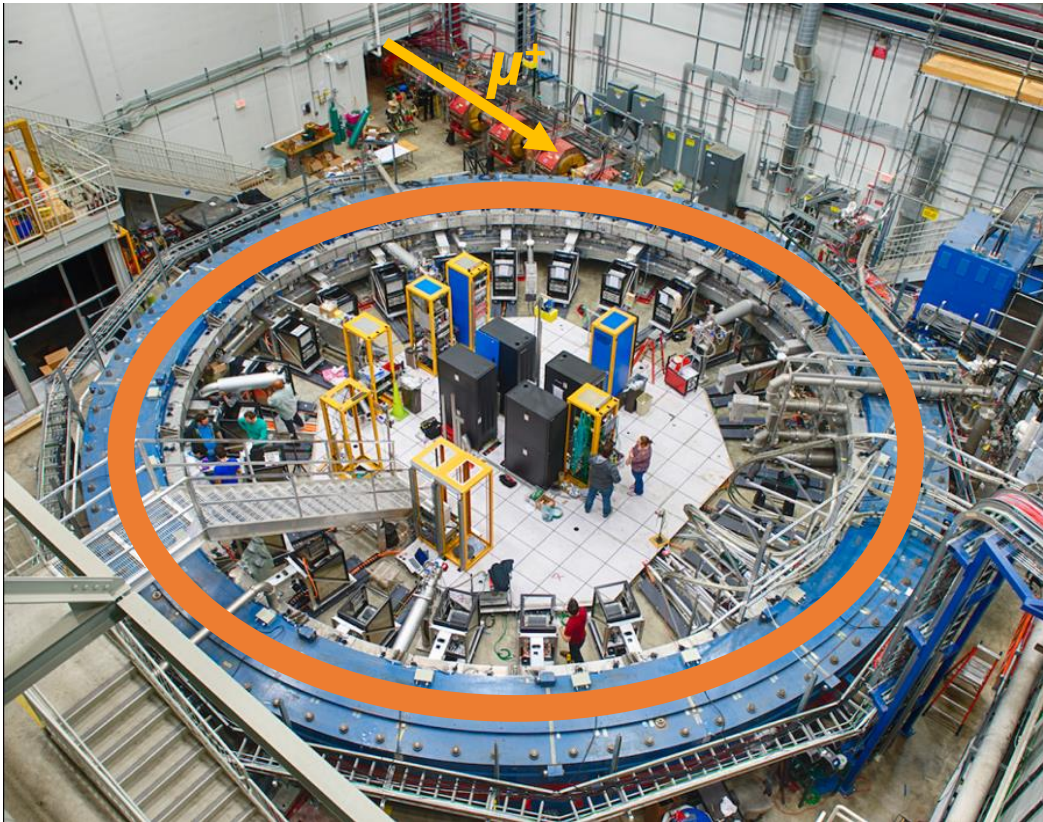


Momentum selective beam line
$$p = 3.094 \frac{\text{GeV}}{c} \pm 2\%$$

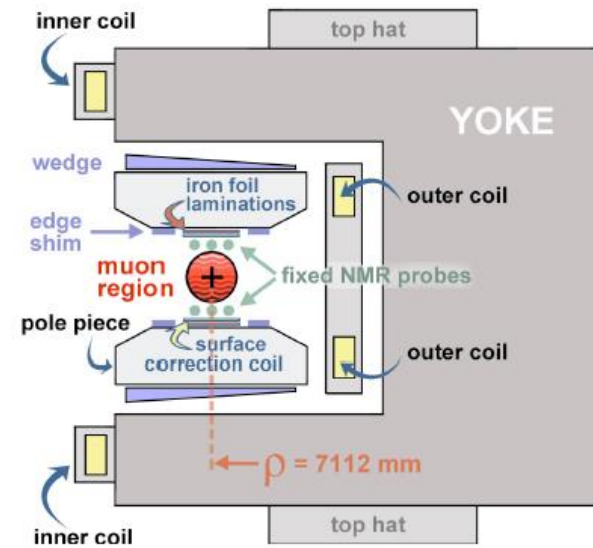
~10,000 μ^+ at 3.1 GeV
every 10 ms and 120 ns bunch length

M. Fertl
R. Reimann

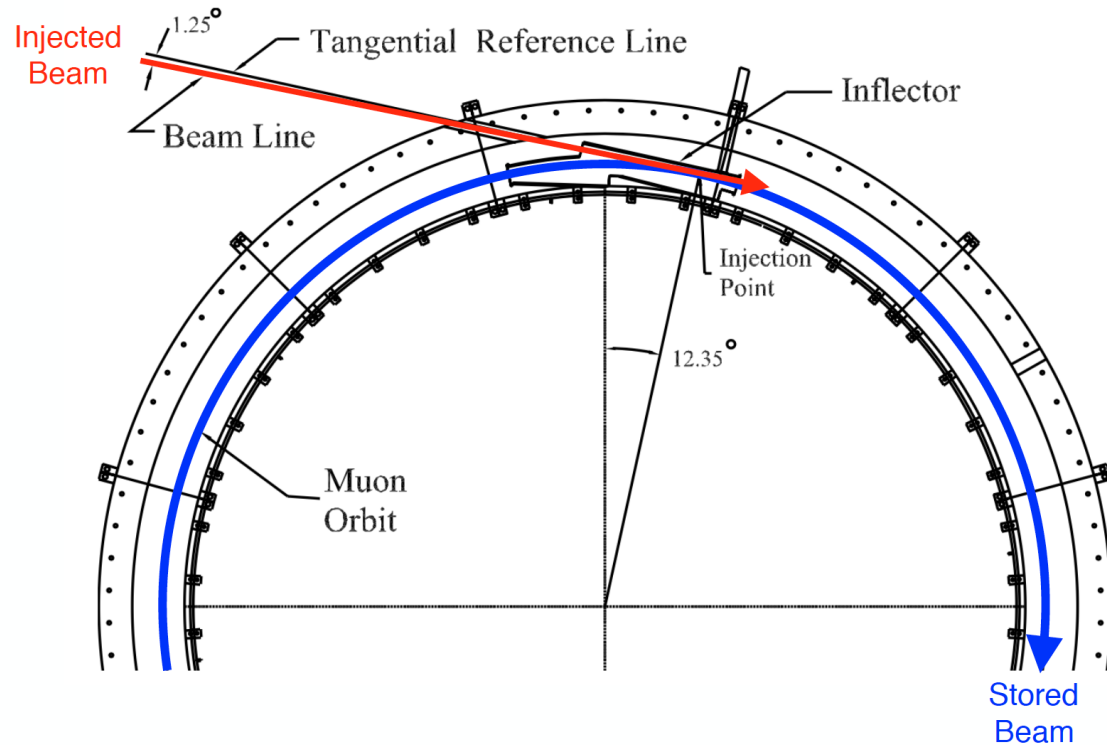
The superconducting storage ring



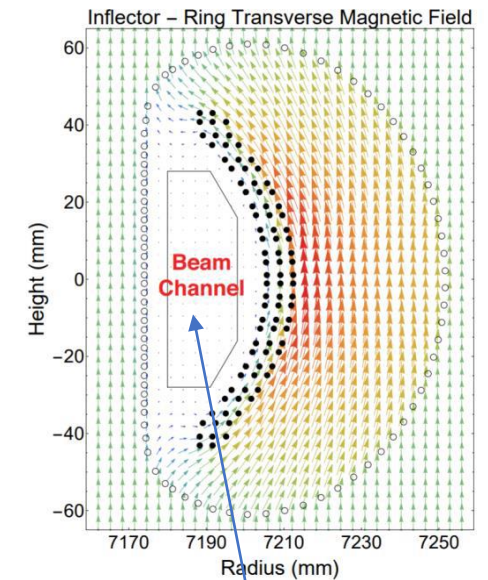
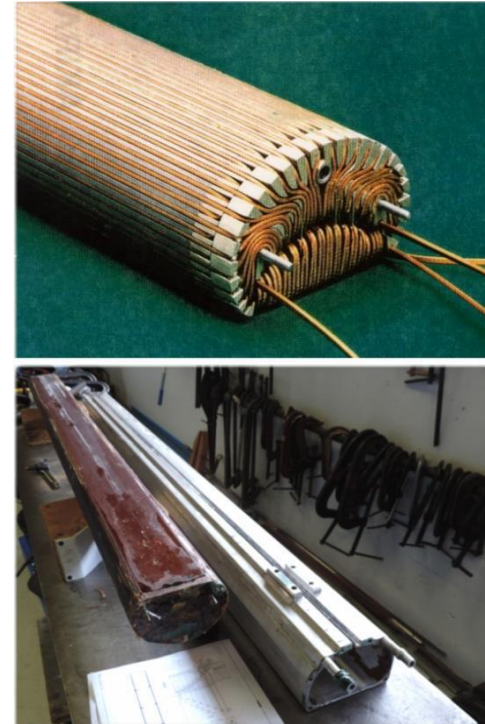
- $p_{\mu}^{magic} = 3.094 \frac{GeV}{c} \pm 0.5\%$
- 3 cryostats with 4 superconducting coils (5300 A)
- 1.45 T vertical magnetic field
- 90 mm muon storage region
- 180 mm gap for vacuum chambers
- muon cyclotron period 149 ns (~ 6.7 MHz)



Beam Injection



- Inflector magnet cancels B field in iron yoke
- Muon can travel straight & enter the ring



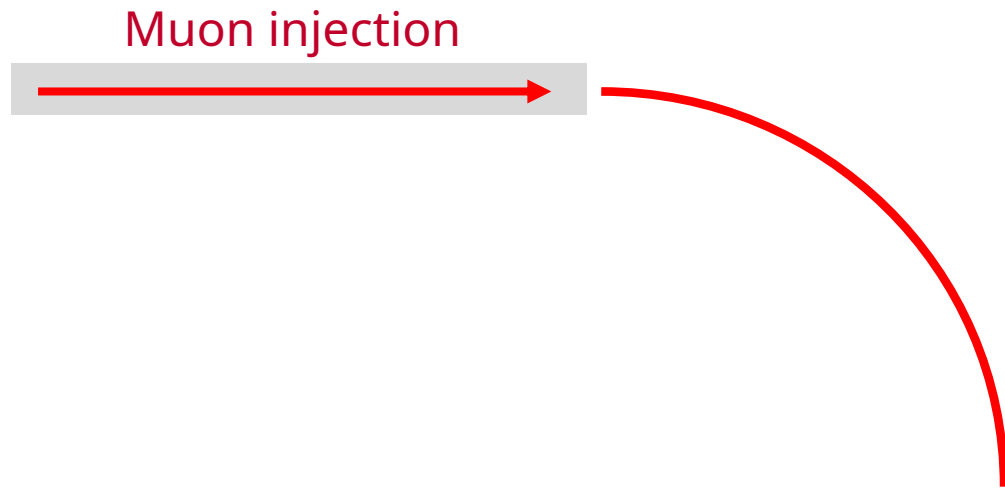
field free region

How to get the beam onto storage orbit?

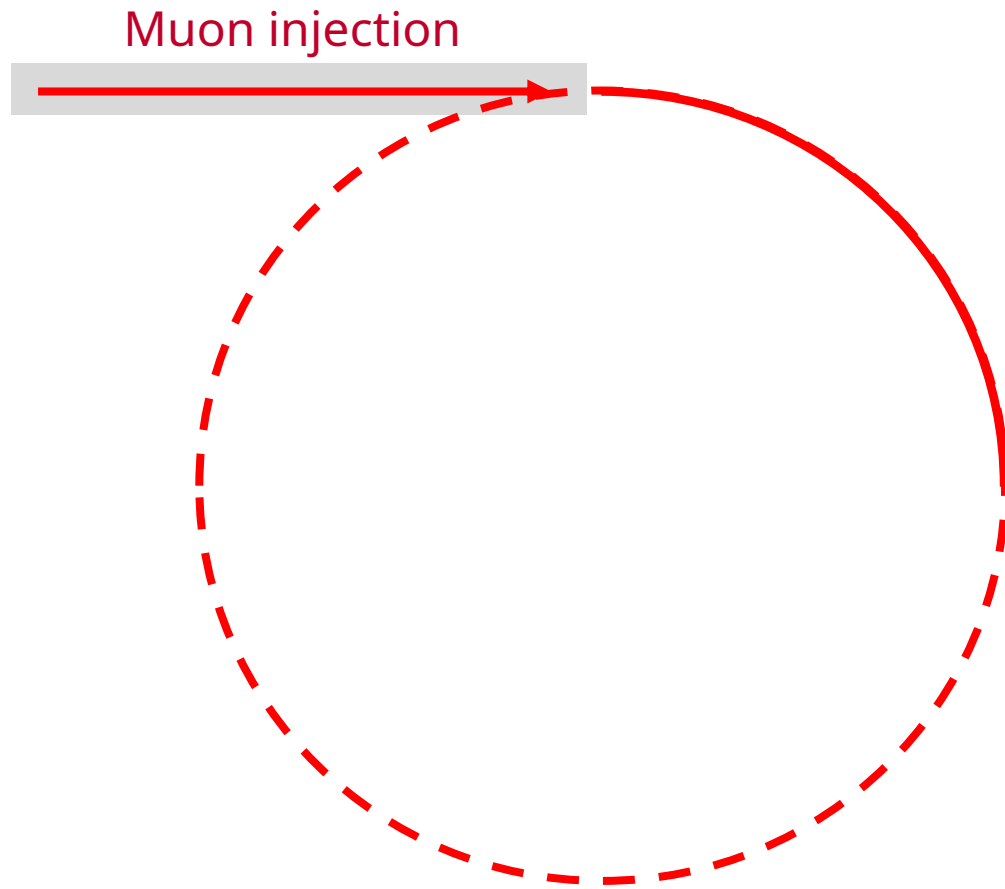
Muon injection



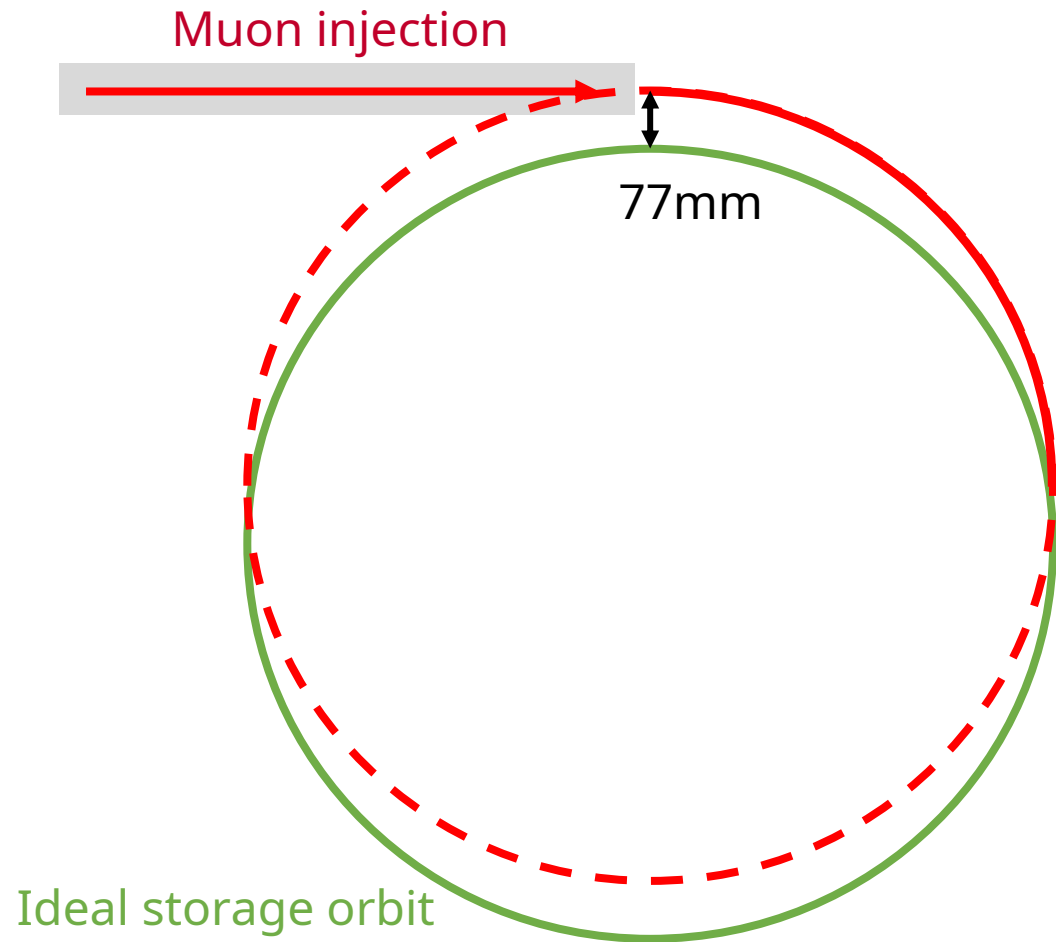
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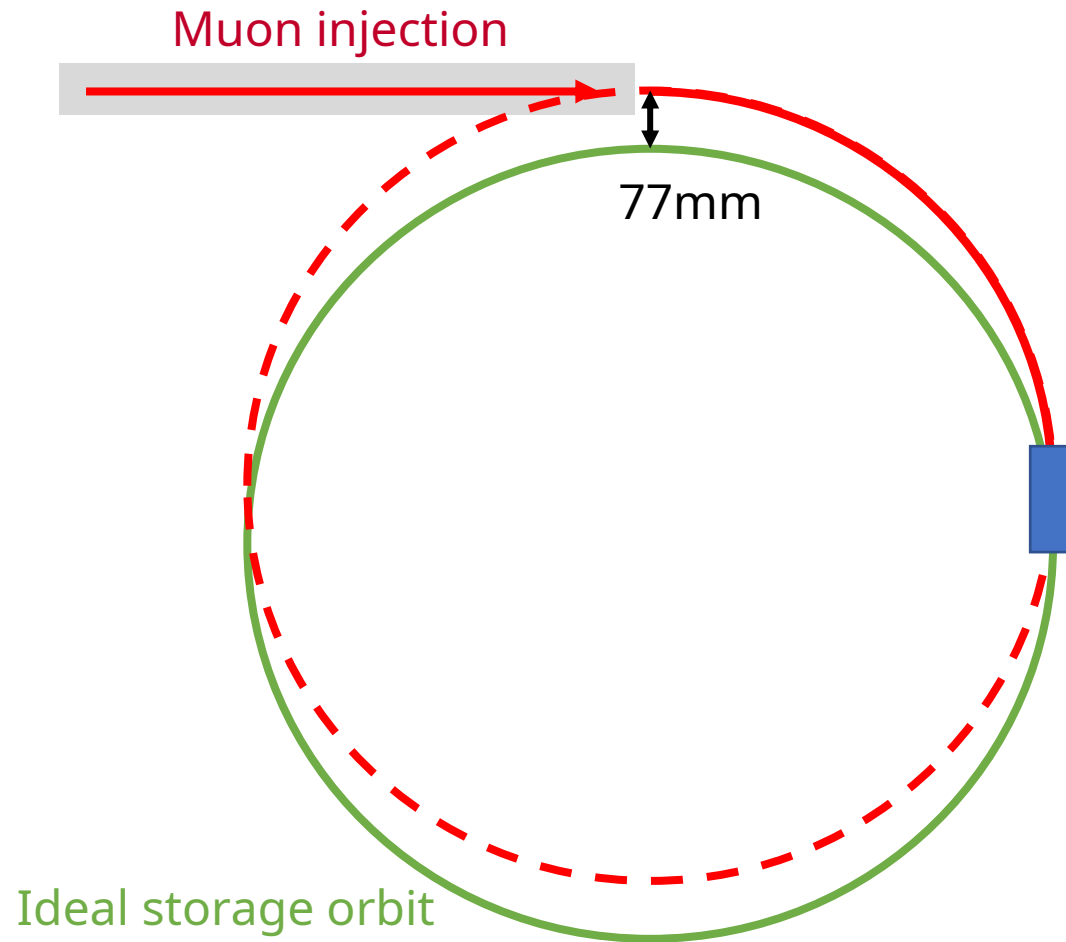
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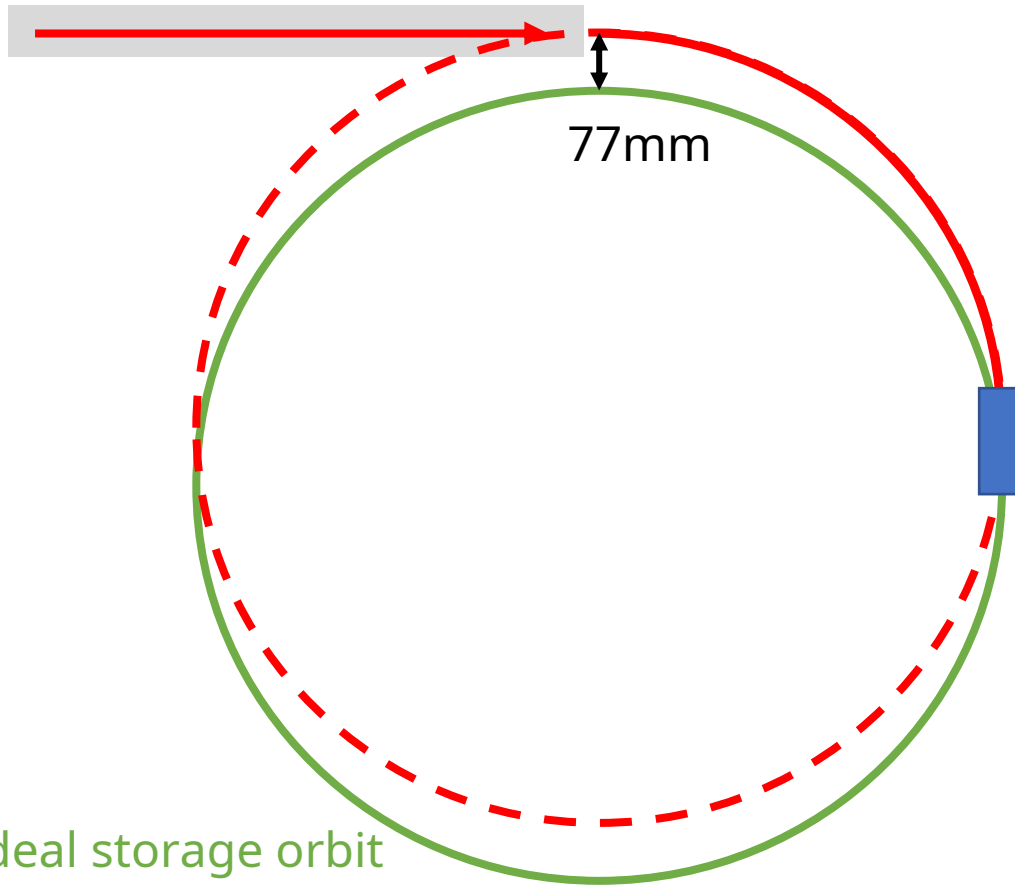


How to get the beam onto storage orbit?



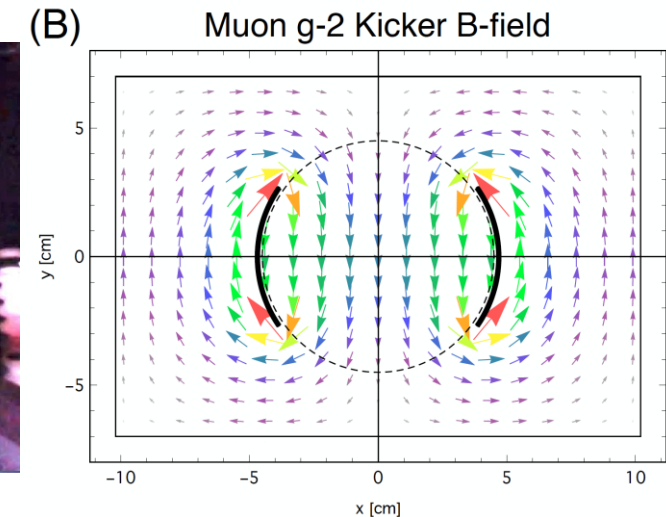
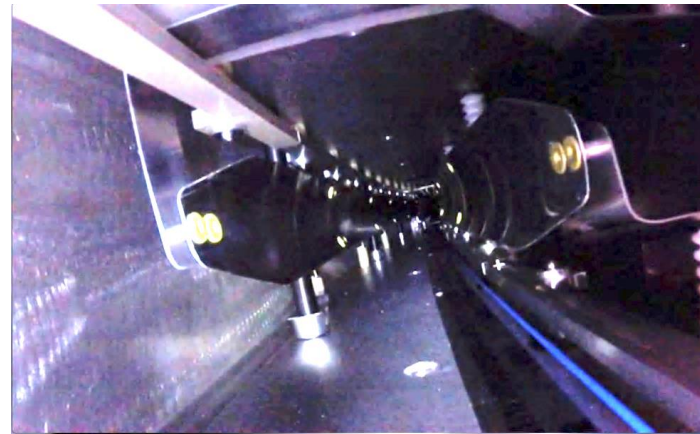
How to get the beam onto storage orbit?

Muon injection

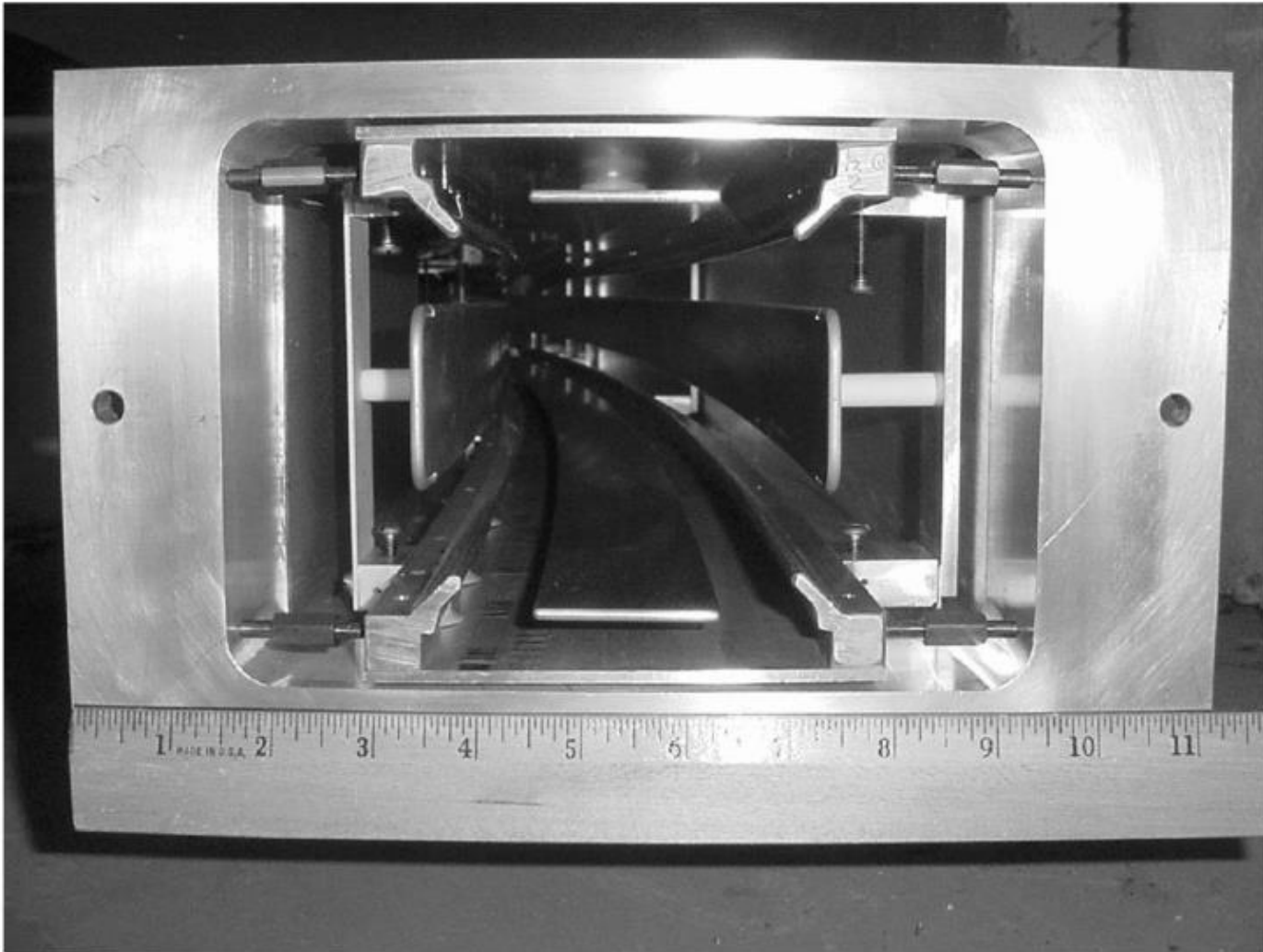


Ideal storage orbit

- Change field locally by 2% within ~ 150 ns
- 3 pairs of plates at roughly 90°
- Apply HV pulse at 4700 A into $\sim 12.5 \Omega$ in 150 ns

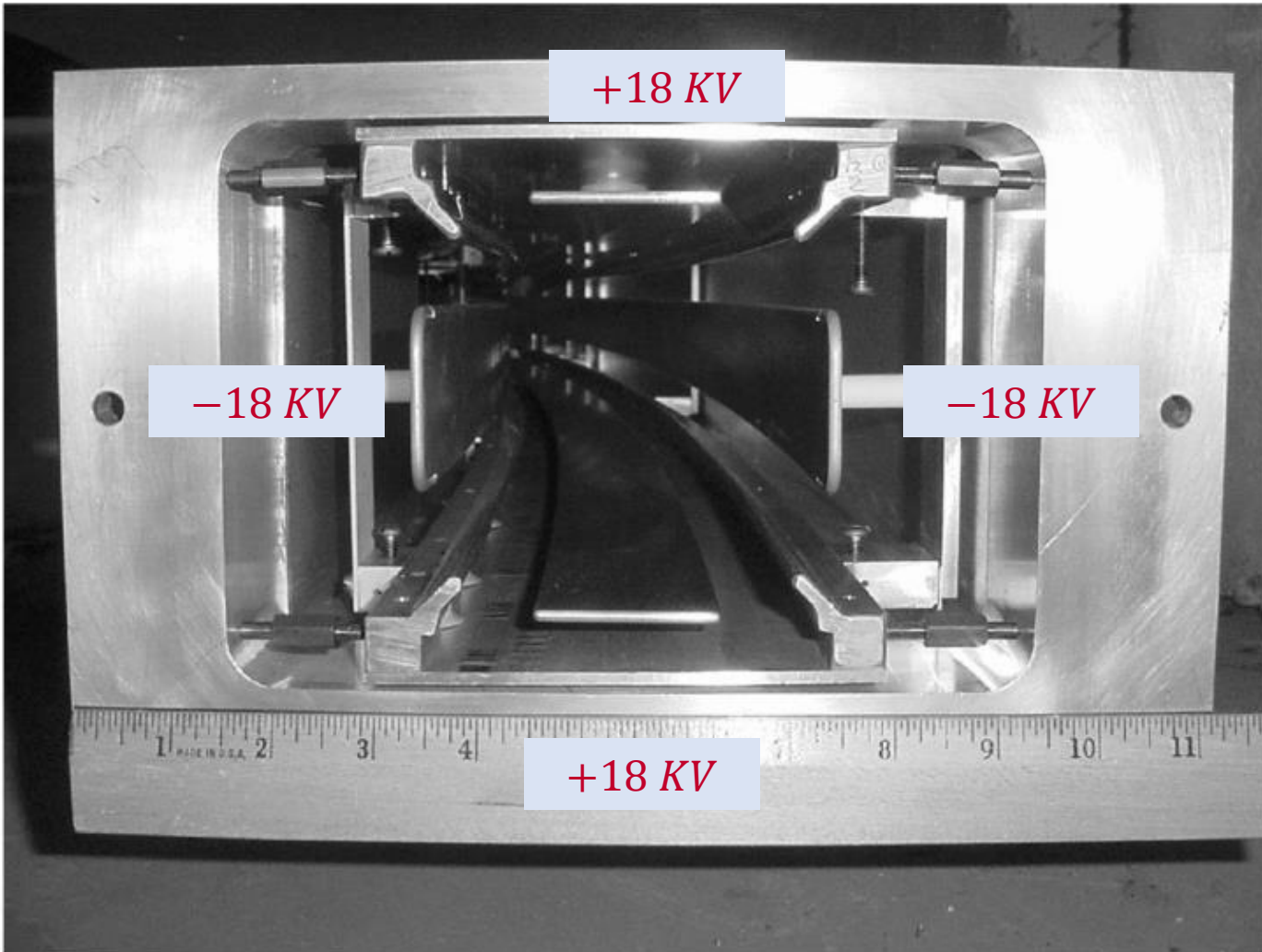


Keeping the muons stored



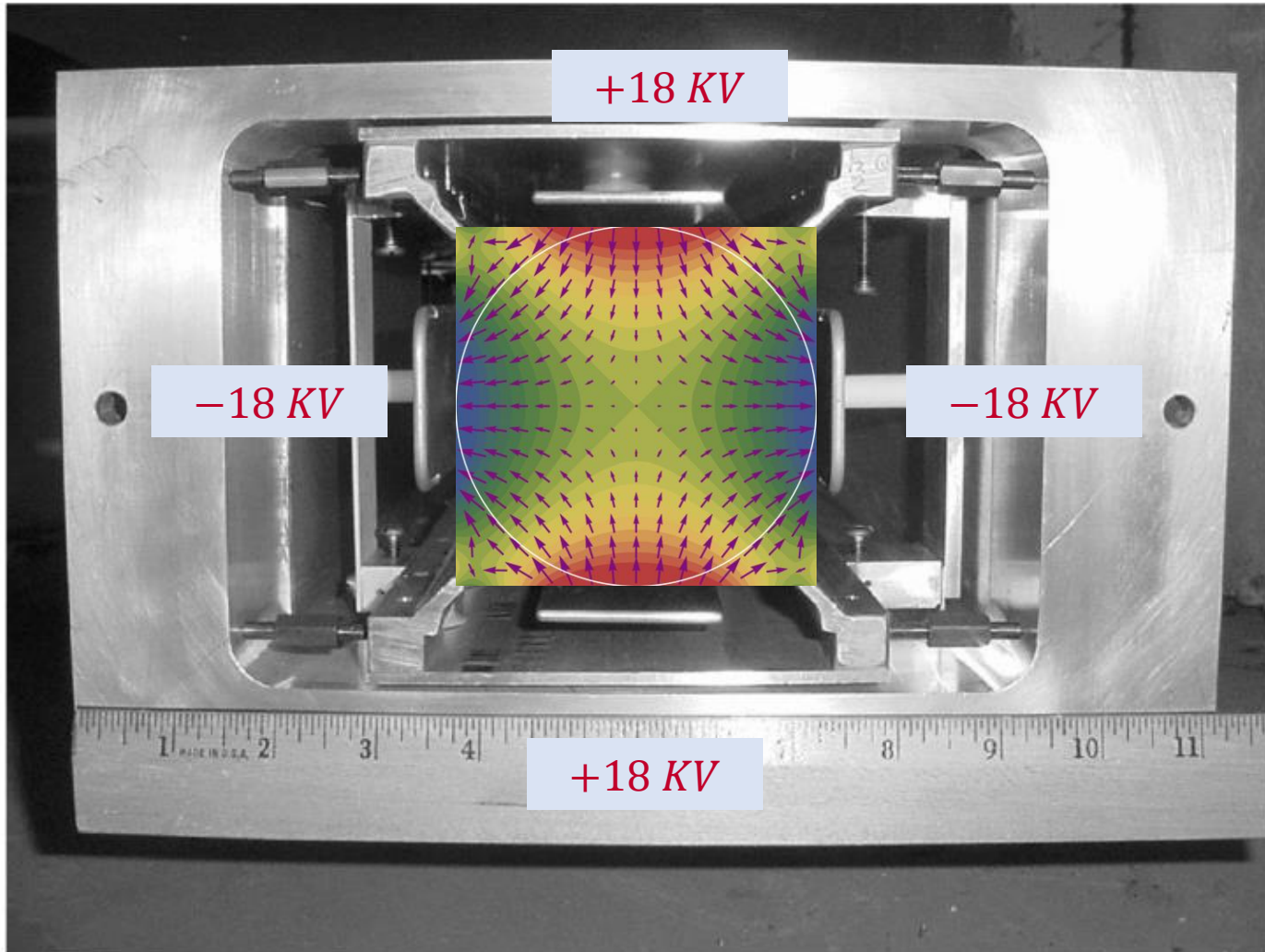
- At magic momentum electric fields have a very small impact on ω_a

Keeping the muons stored



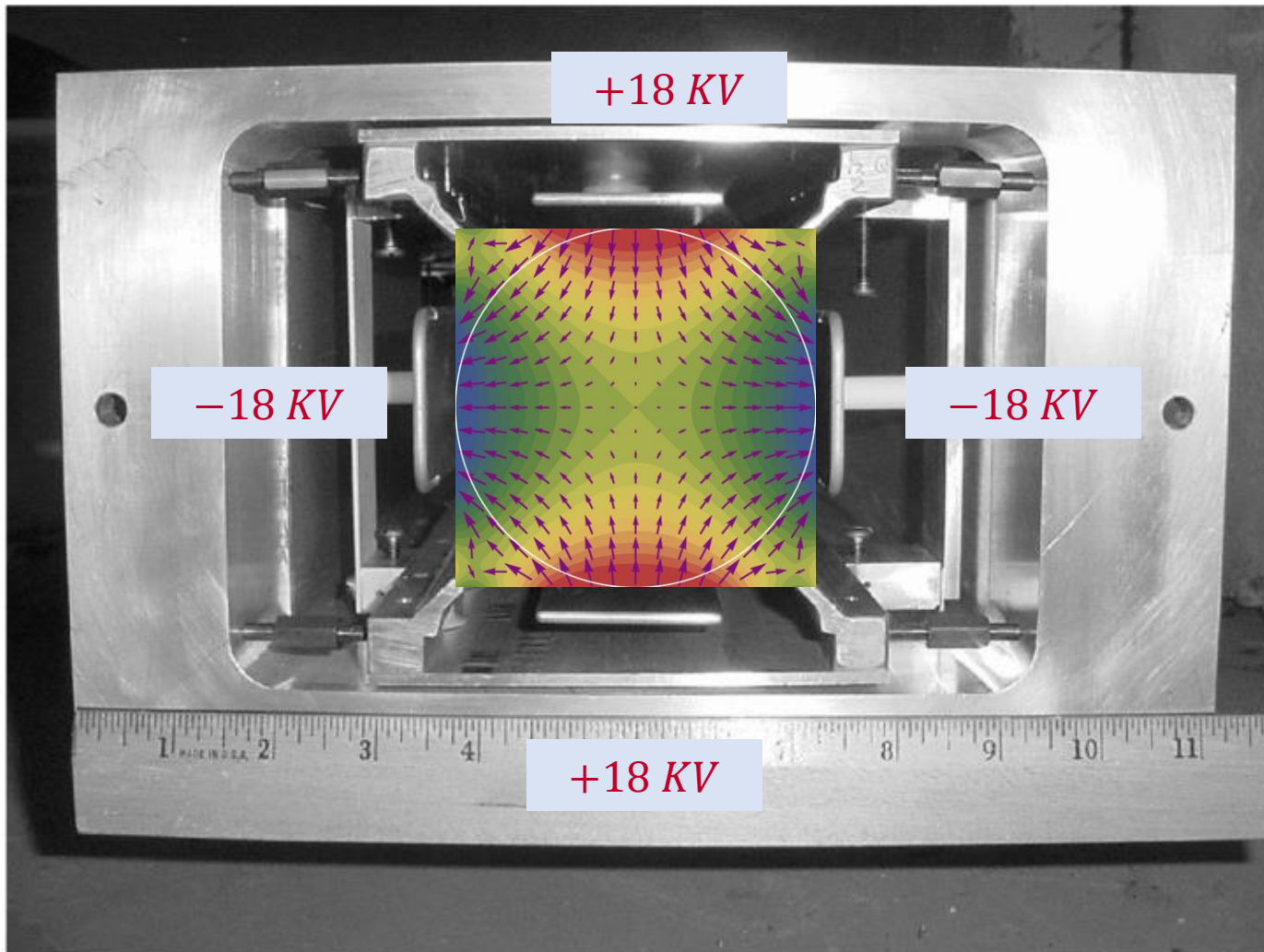
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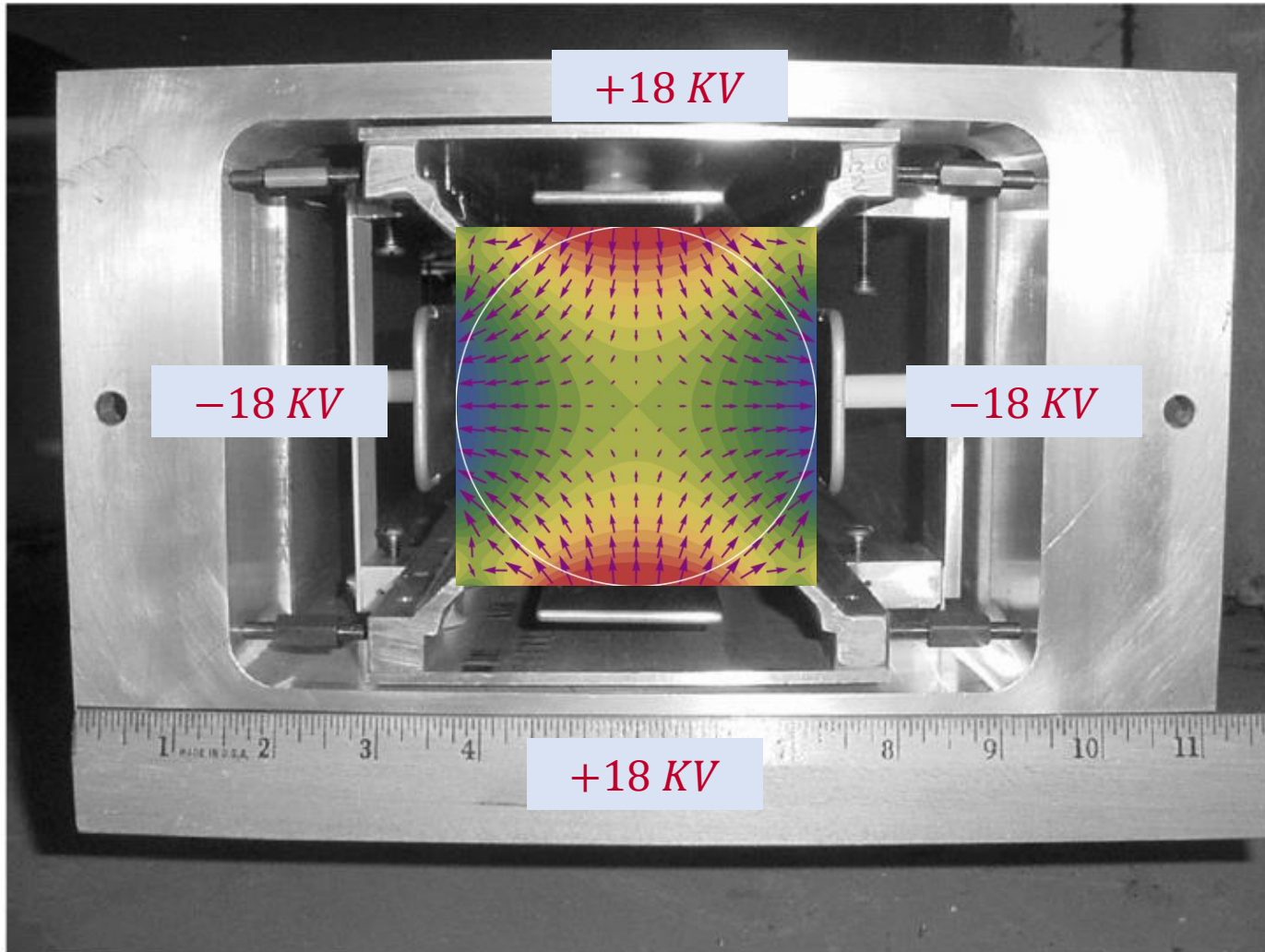
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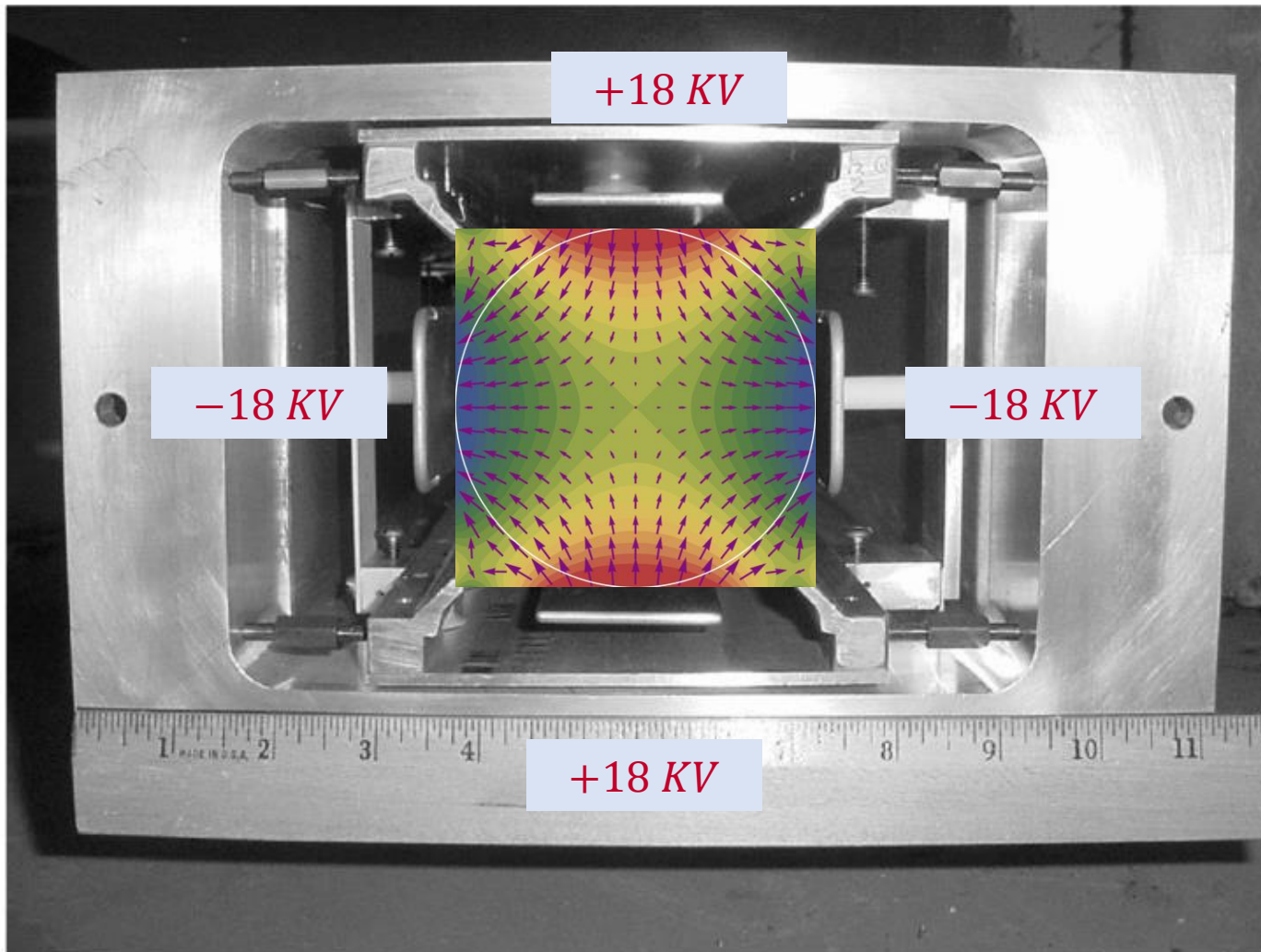
- At magic momentum electric fields have a very small impact on ω_a
- Electrostatic quadrupoles focus beam vertically
- Electrostatic quadrupoles defocus beam radially
- Magnetic field focus beam radially
→ Complex beam dynamics

Keeping the muons stored



- At magic momentum electric fields have a very small impact on ω_a
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- Quasi-penning trap cover 43% of the ring

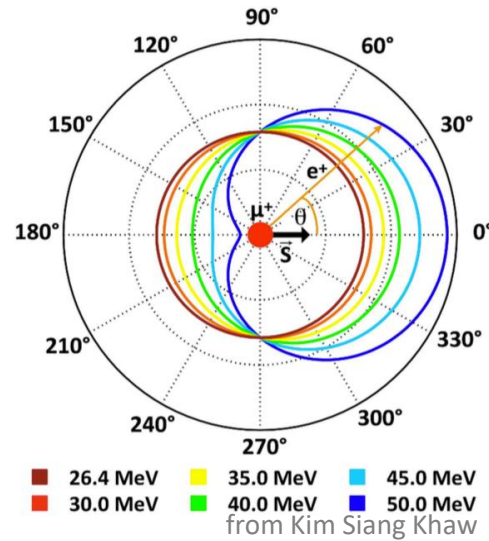
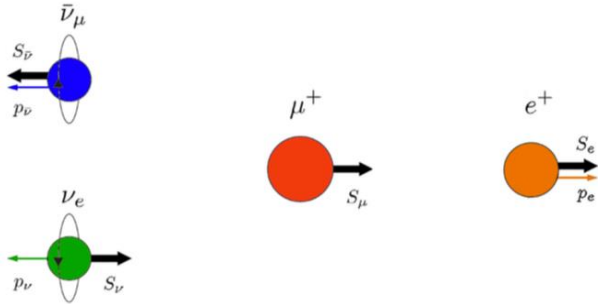
Keeping the muons stored



- At magic momentum electric fields have a very small impact on ω_a
- Electrostatic quadrupoles focus beam vertically
- Electrostatic quadrupoles defocus beam radially
- Magnetic field focus beam radially
→ Complex beam dynamics
- Quasi-penning trap cover 43% of the ring
- Pulsed “electrostatic” quadrupoles

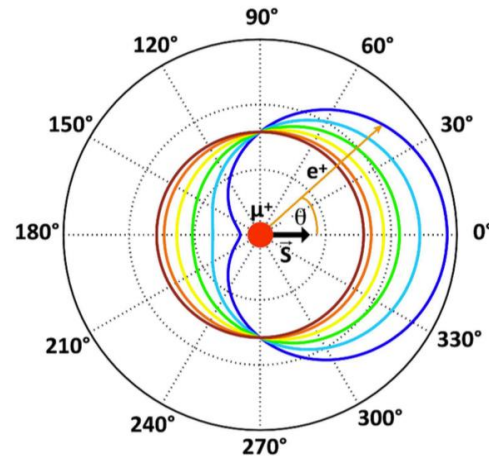
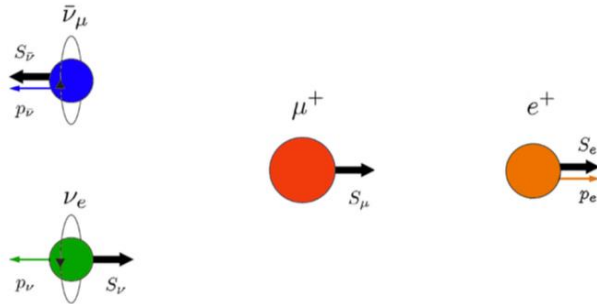
Spin projection detection

muon rest frame



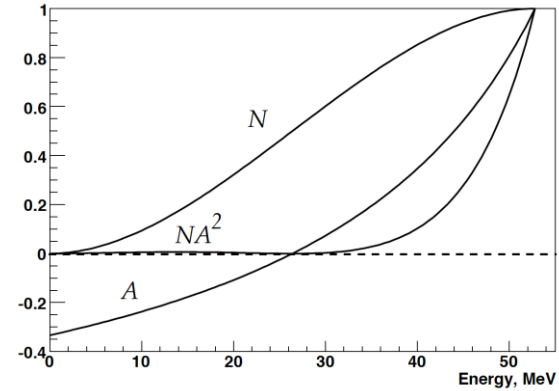
Spin projection detection

muon rest frame



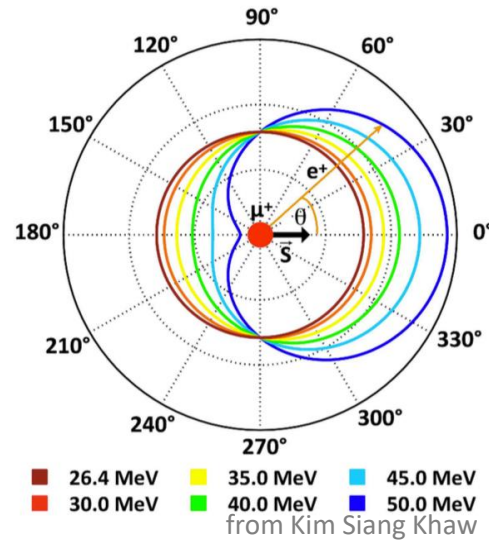
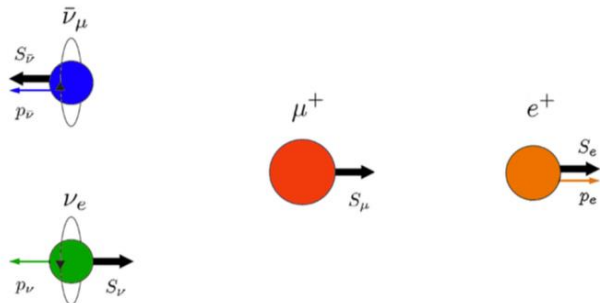
- 26.4 MeV
 - 30.0 MeV
 - 35.0 MeV
 - 40.0 MeV
 - 45.0 MeV
 - 50.0 MeV
- from Kim Siang Khaw

rest frame

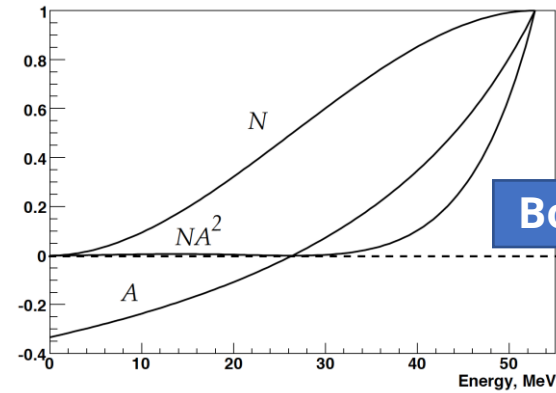


Spin projection detection

muon rest frame

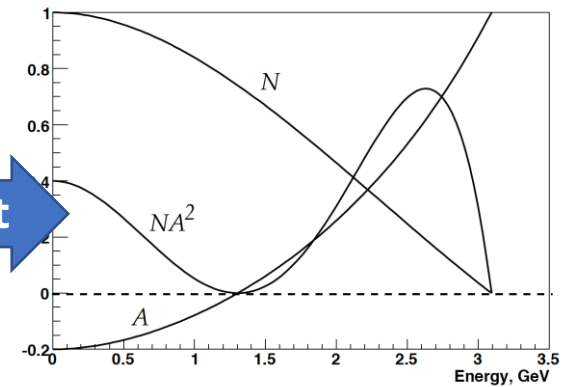


rest frame



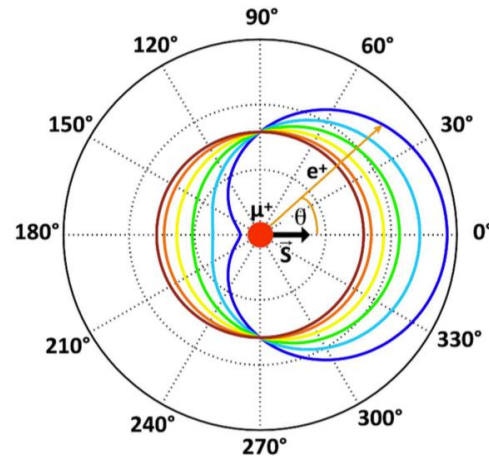
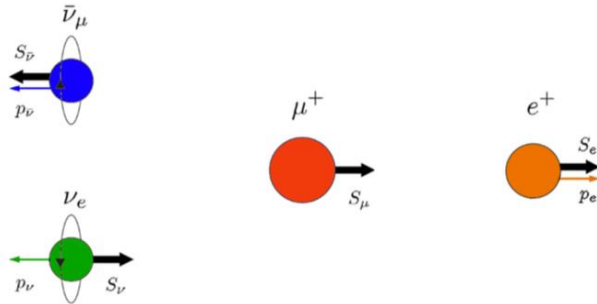
Boost

laboratory frame



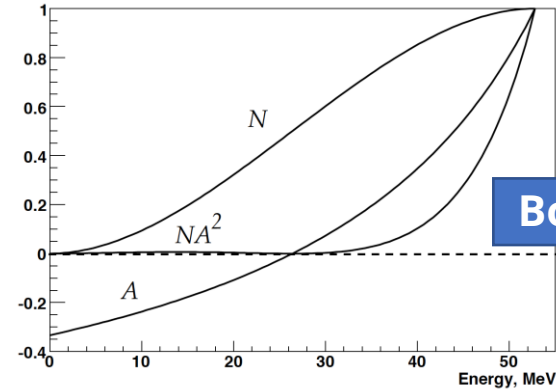
Spin projection detection

muon rest frame

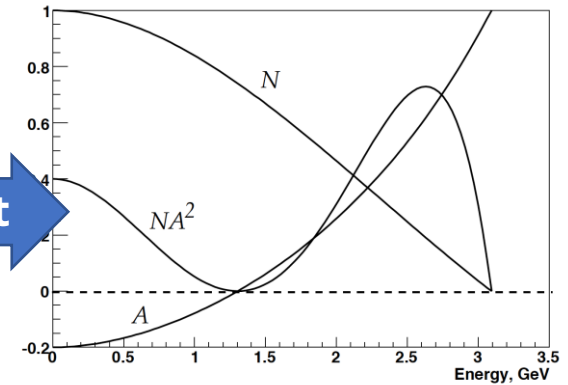


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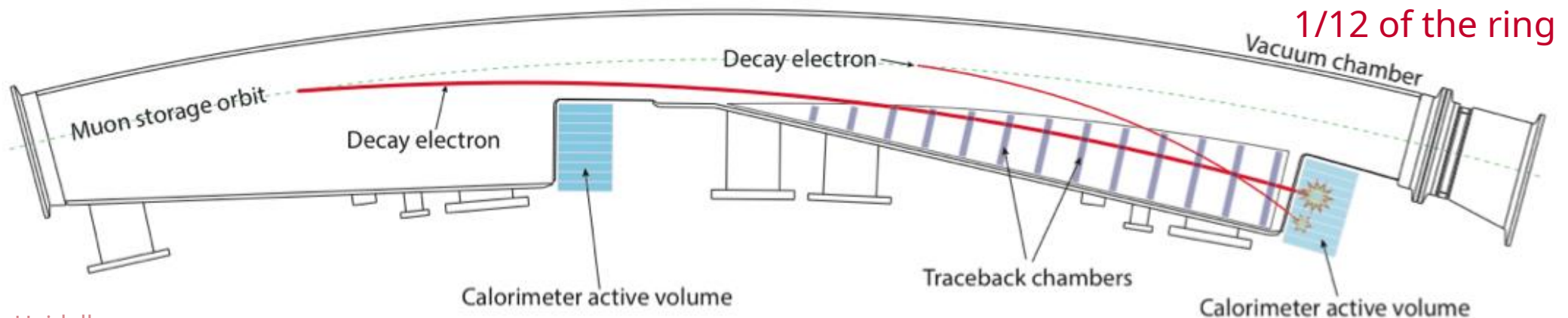
rest frame



laboratory frame



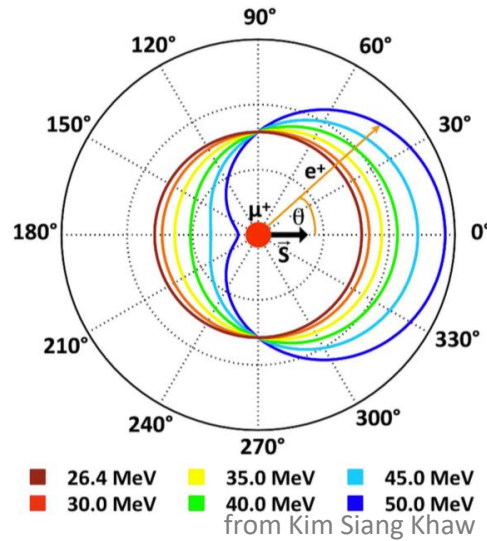
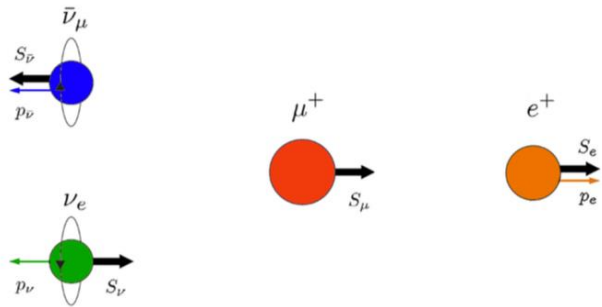
Boost



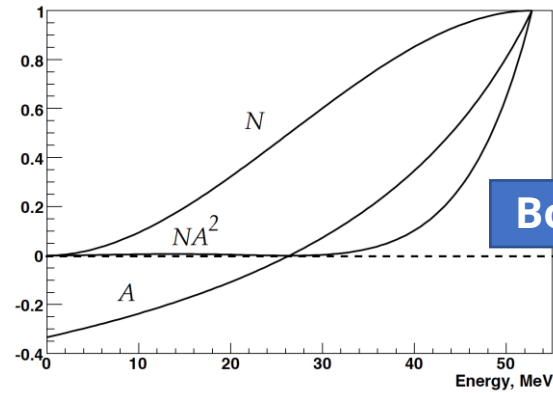
1/12 of the ring

Spin projection detection

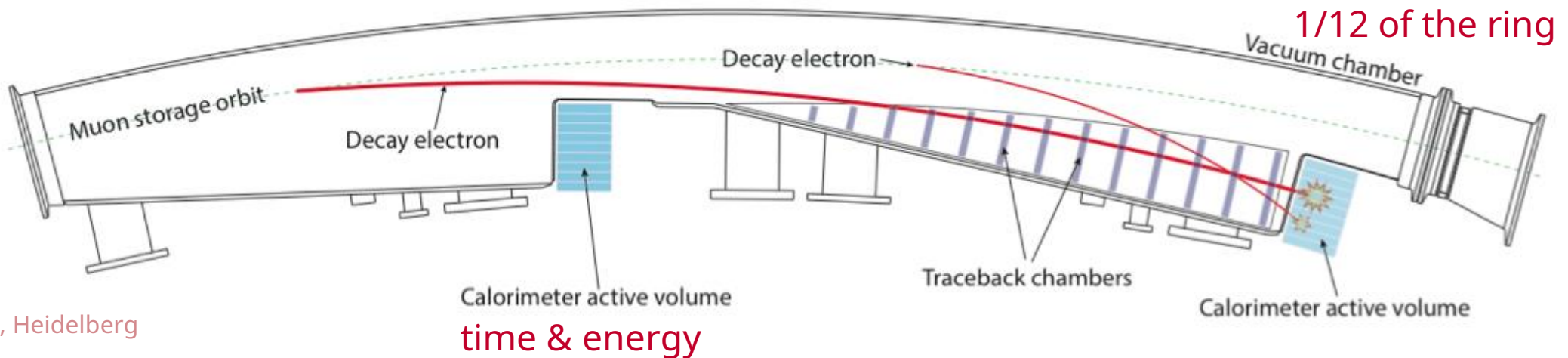
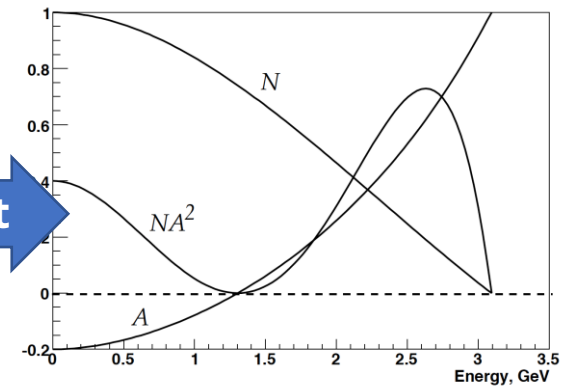
muon rest frame



rest frame

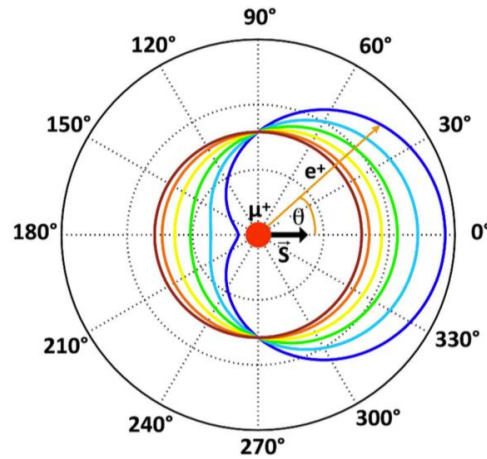
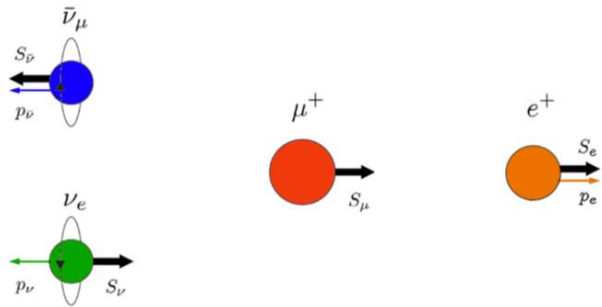


laboratory frame



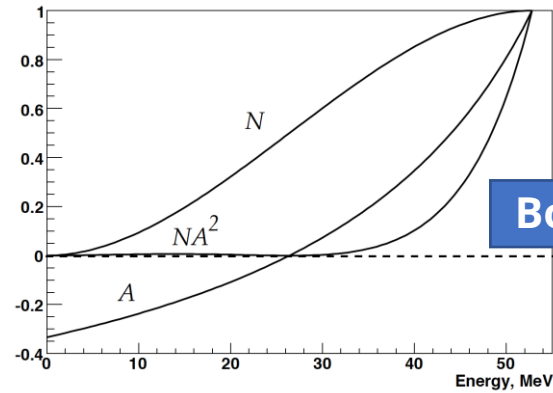
Spin projection detection

muon rest frame

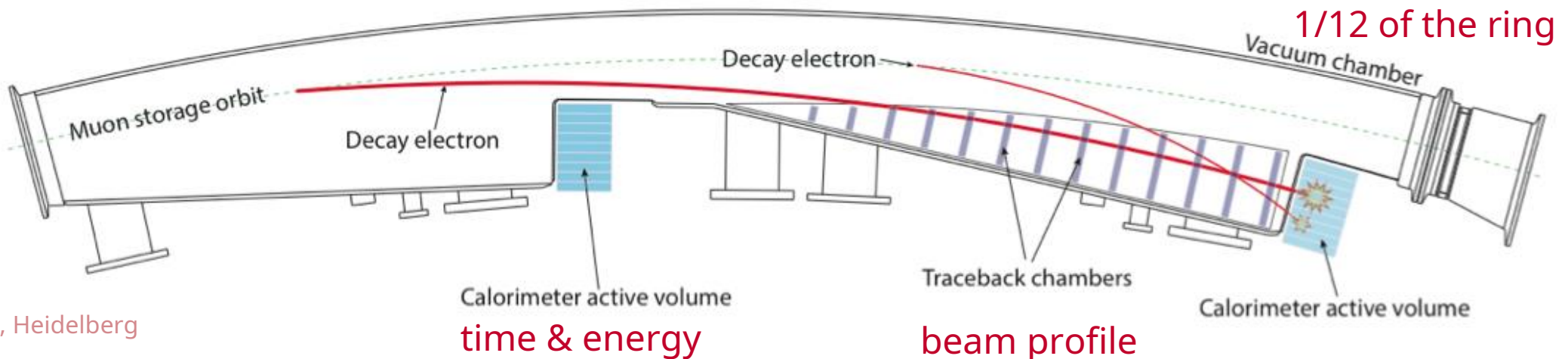
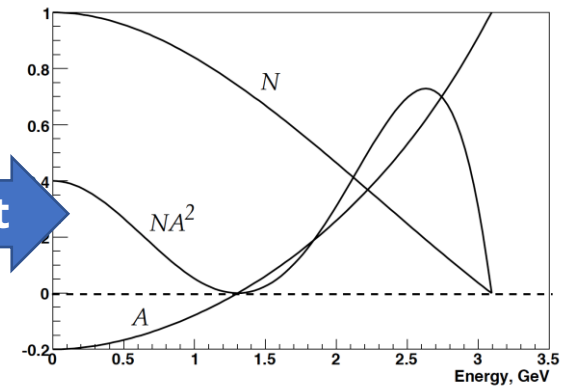


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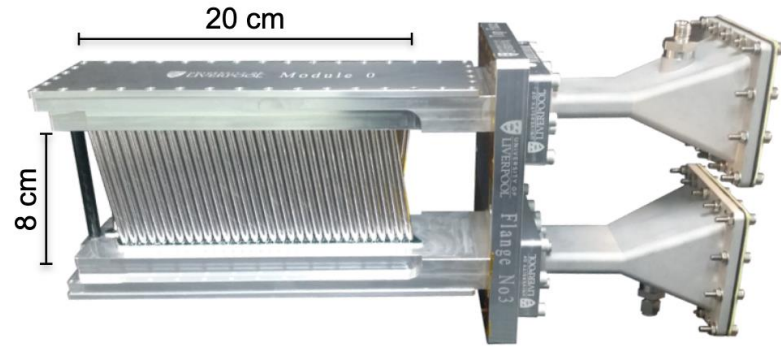
rest frame



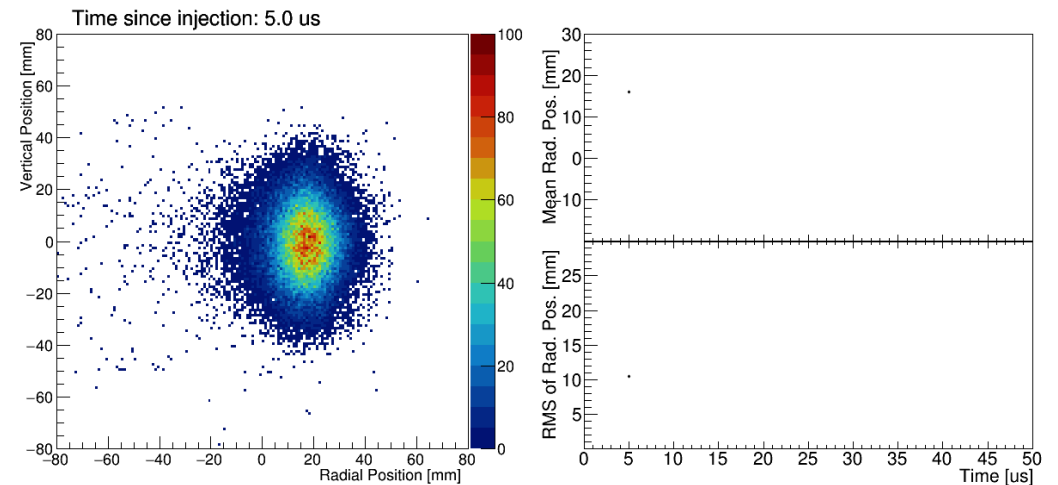
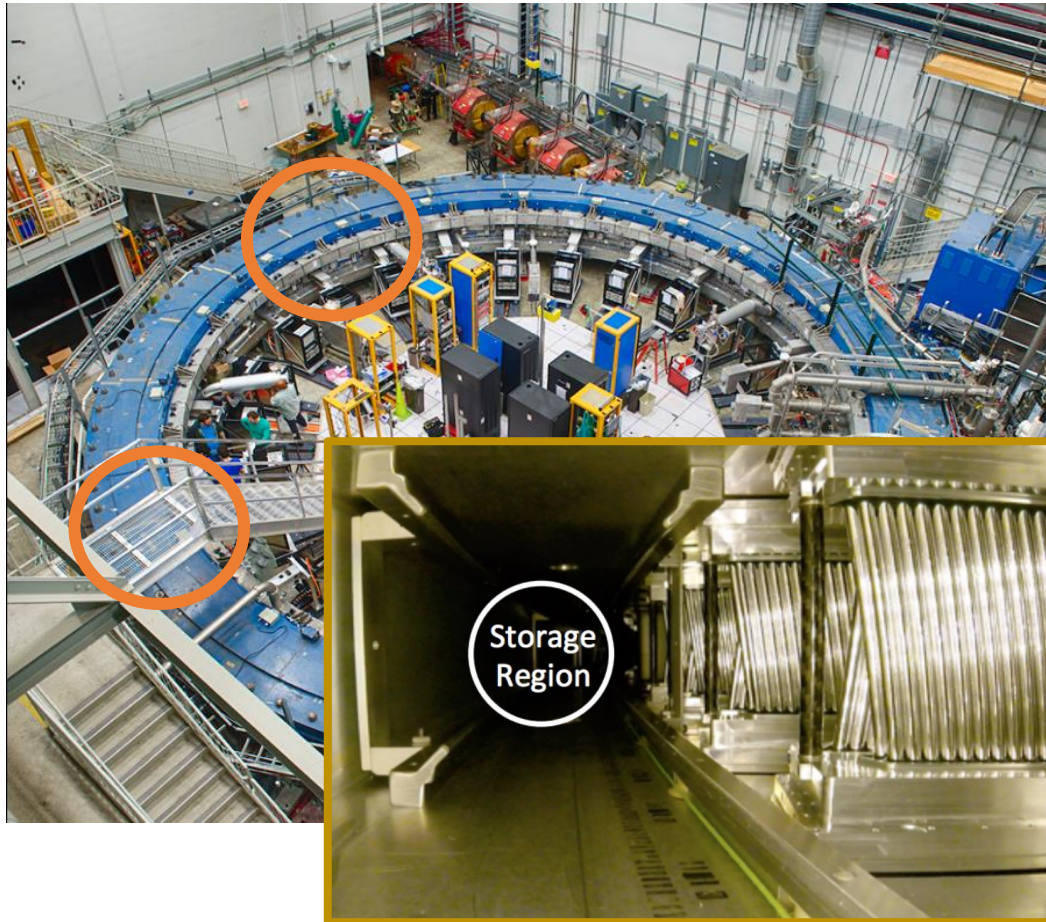
laboratory frame



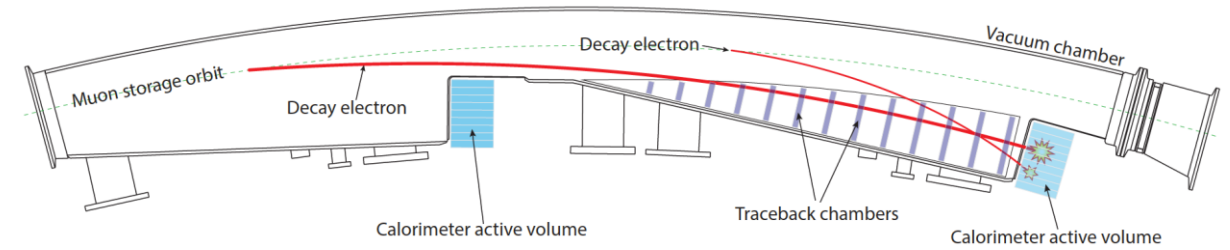
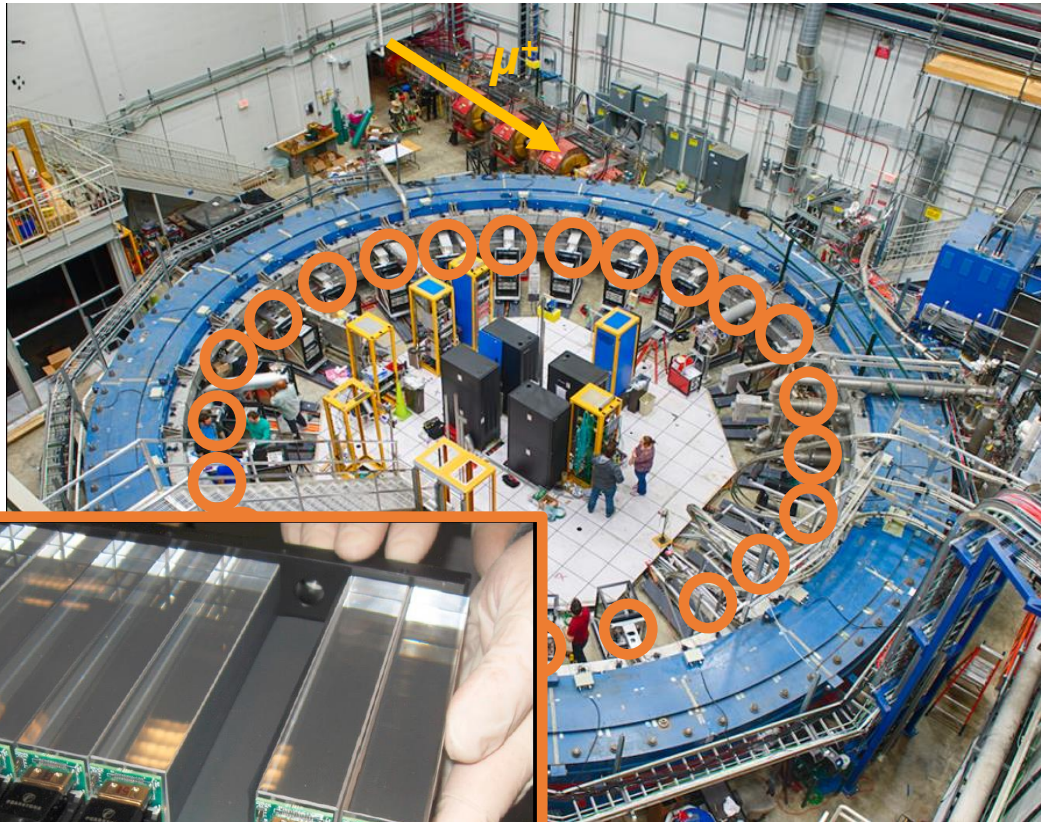
Tracking detectors



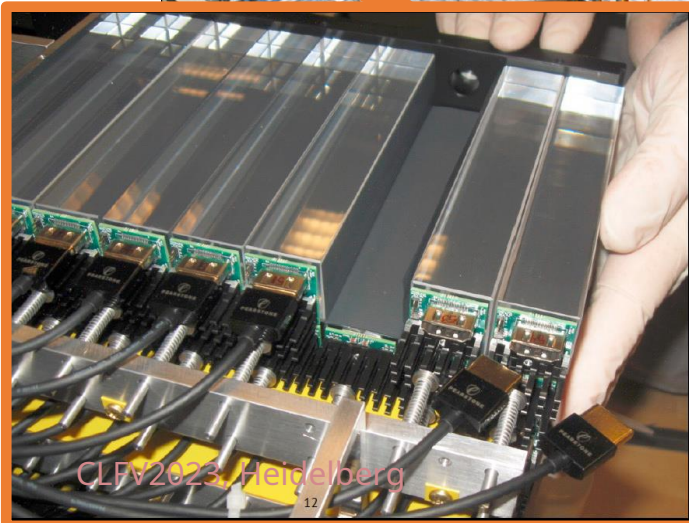
- Two tracking stations, each with 8 modules
- 128 gas-filled straws per module
- Determine e^+ trajectory to decay position and extrapolate to find muon beam distribution!
- Input for beam dynamics simulations



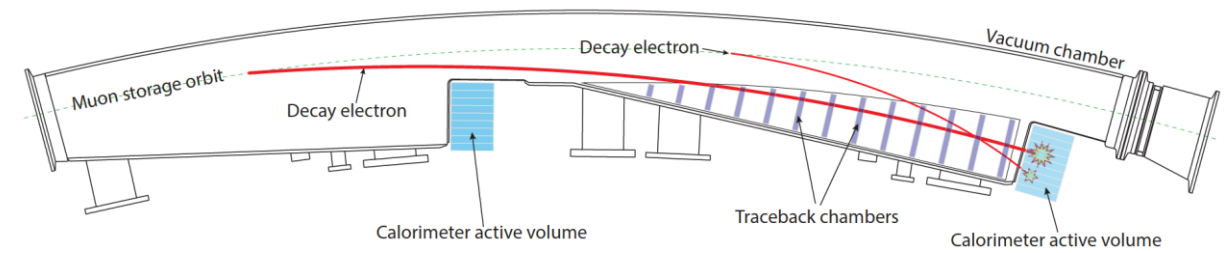
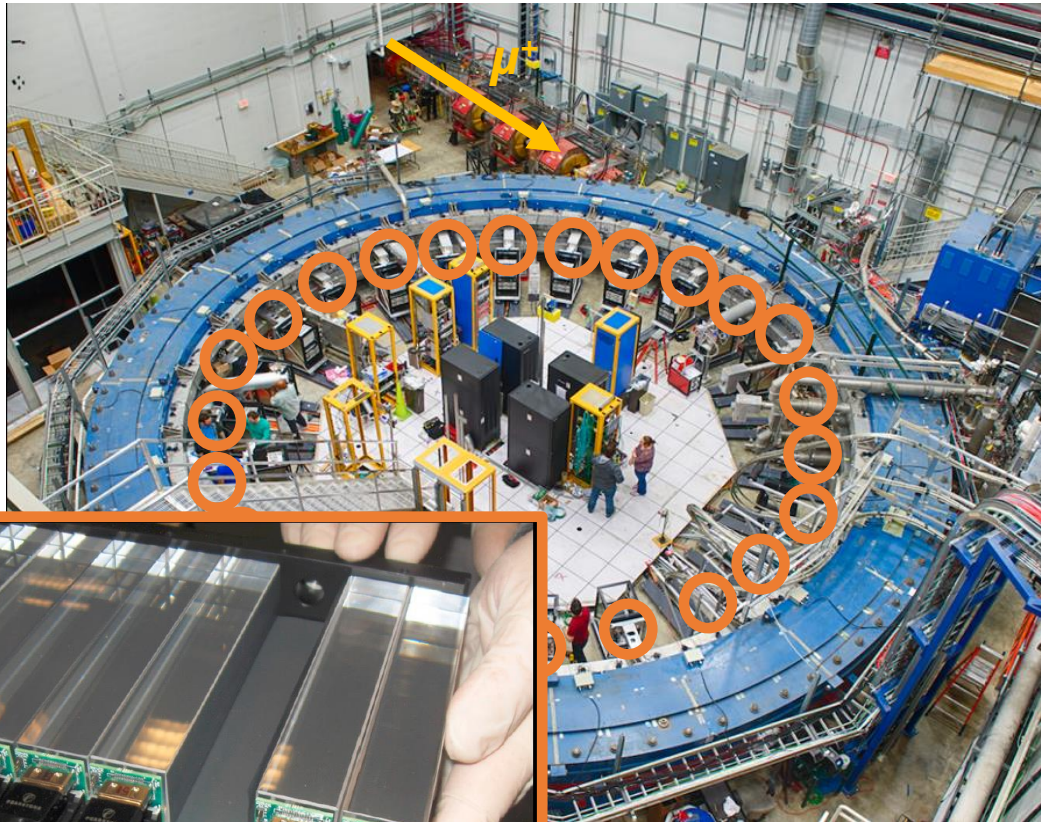
Positron detection



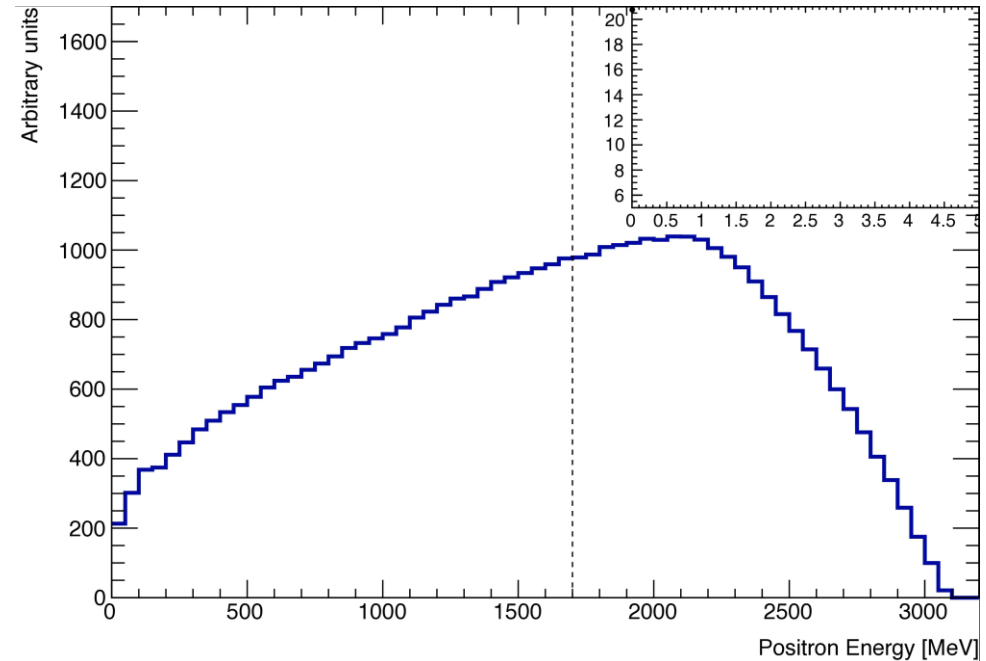
- 24 calorimeter stations
- 9 x 6 arrays of PbF₂ crystals (Cherenkov detectors!)
- Individual SiPM readout boards



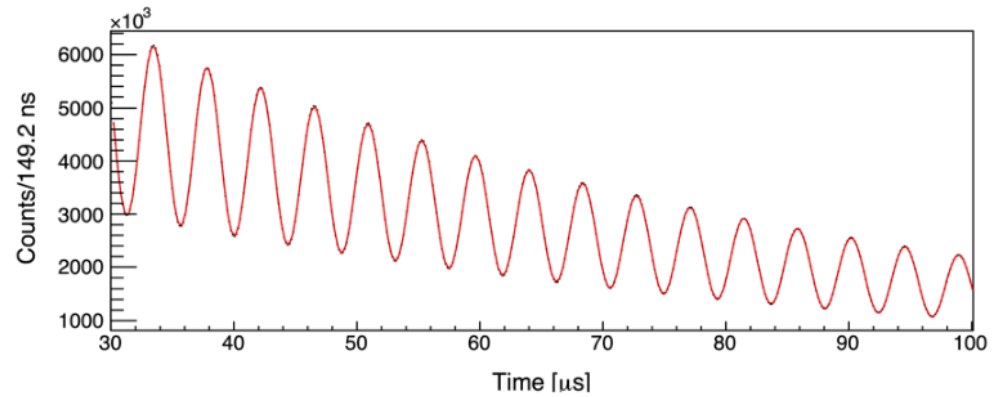
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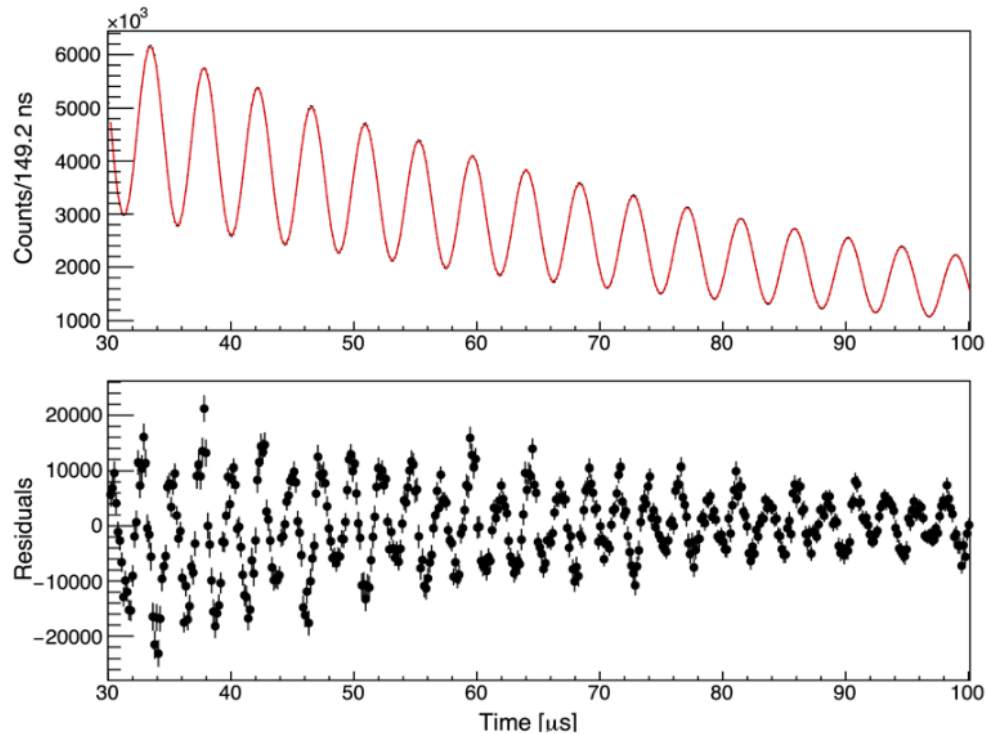


Fitting the “wiggle” plot



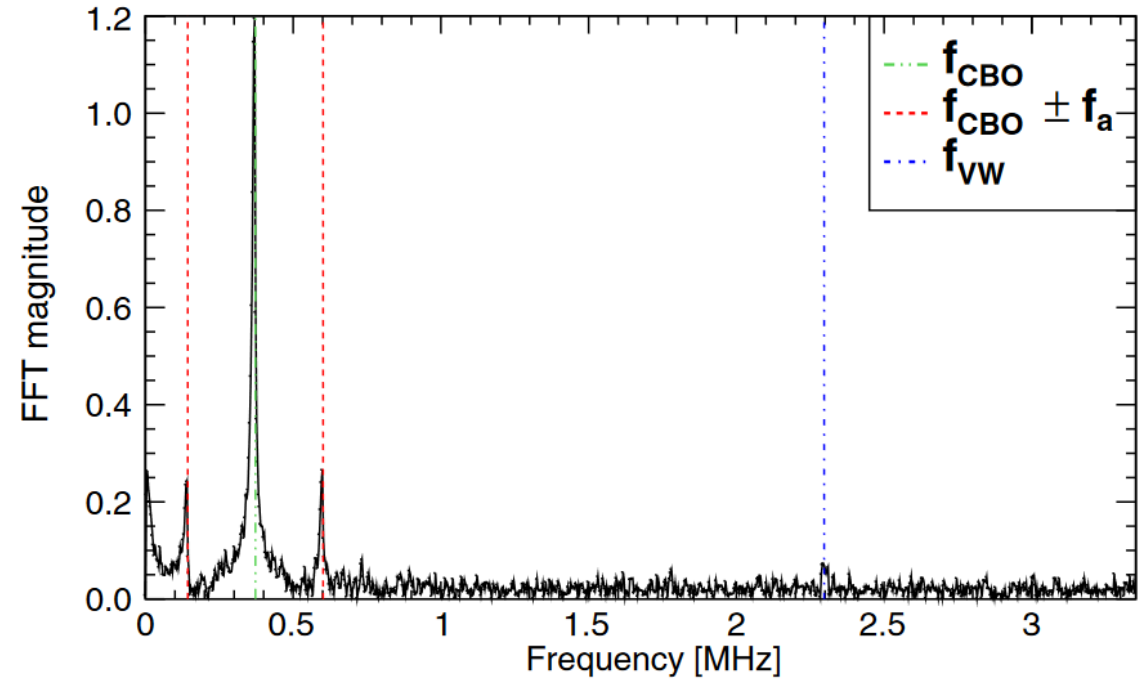
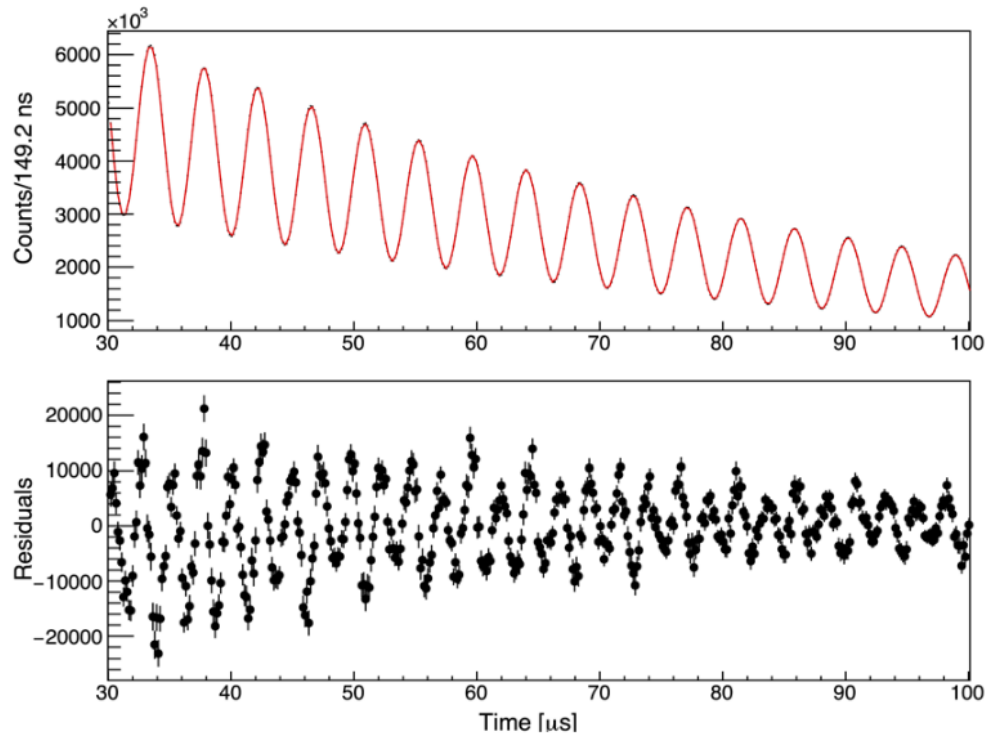
Fitting the “wiggle” plot

$$f(t) \propto N_0 e^{-\frac{t}{\tau}} [\langle N \rangle_{\text{thresh}} + \langle A \rangle_{\text{thresh}} \cos(\omega_a t - \langle \phi \rangle_{\text{thresh}})]$$



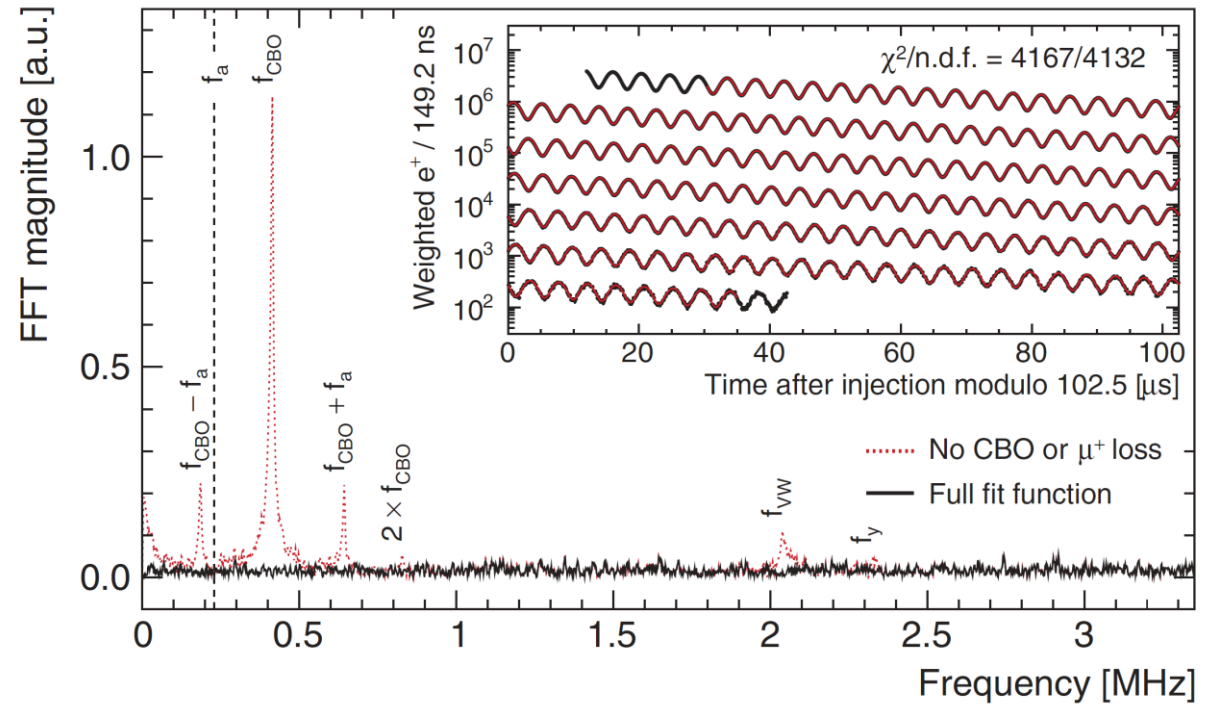
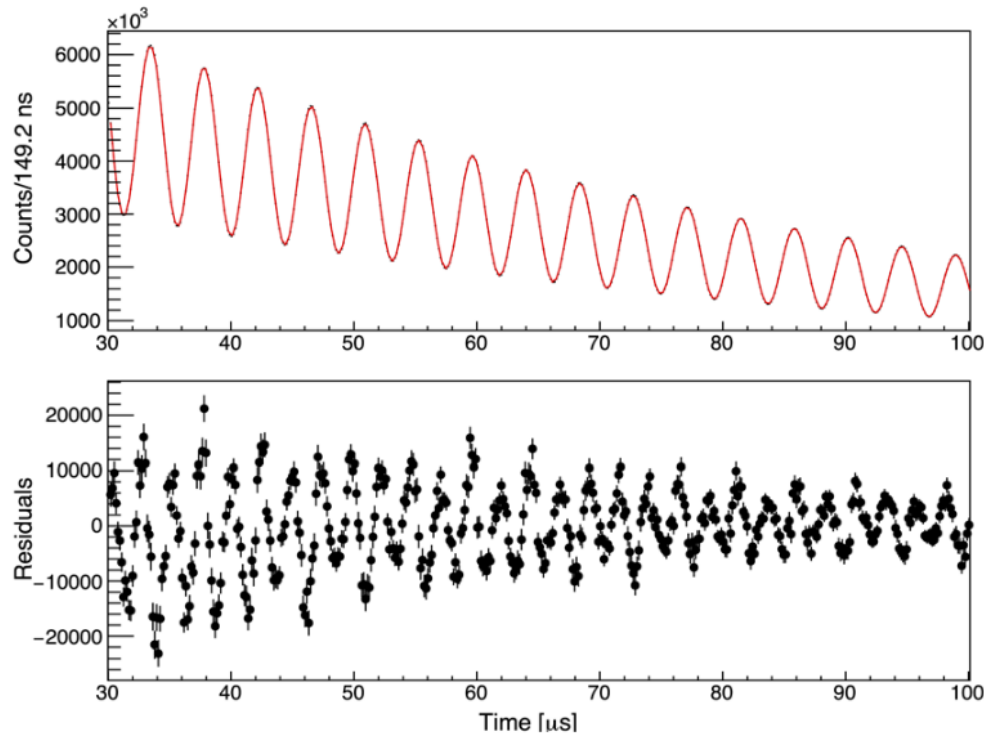
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$$f(t) \propto N_0 e^{-\frac{t}{\gamma\tau}} [\langle N \rangle_{\text{thresh}} + \langle A \rangle_{\text{thresh}} \cos(\omega_a t - \langle \phi \rangle_{\text{thresh}})]$$



Fitting the “wiggle” plot

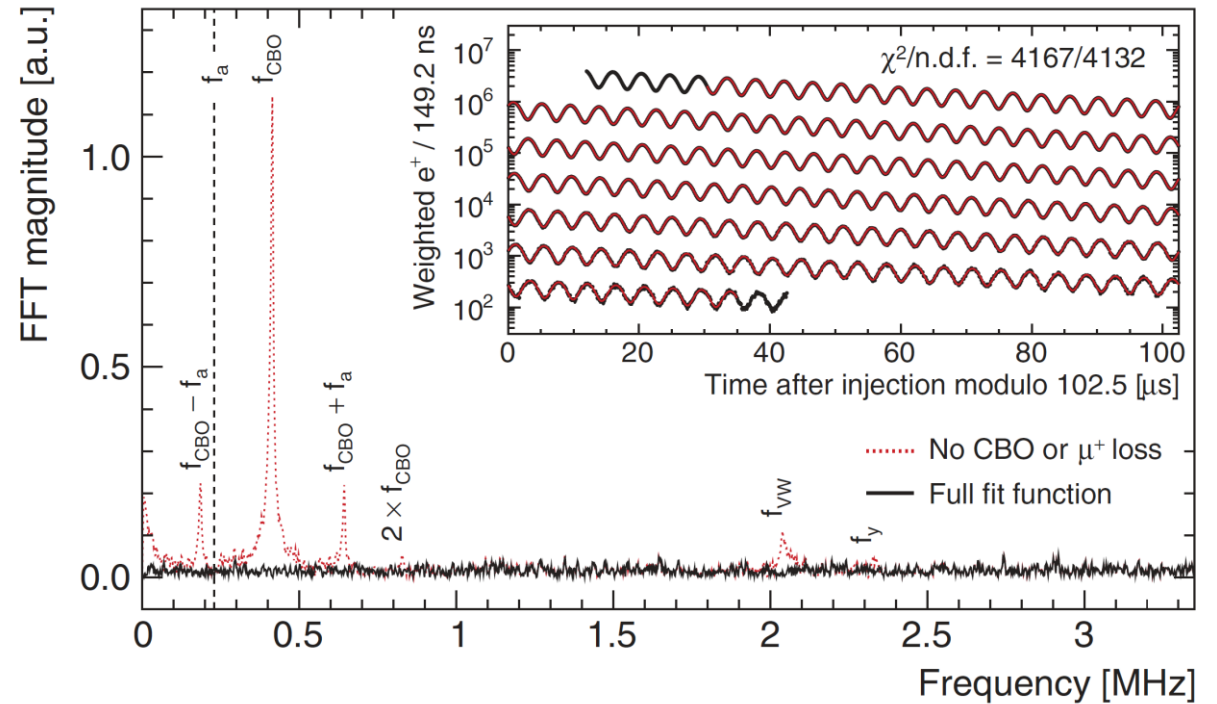
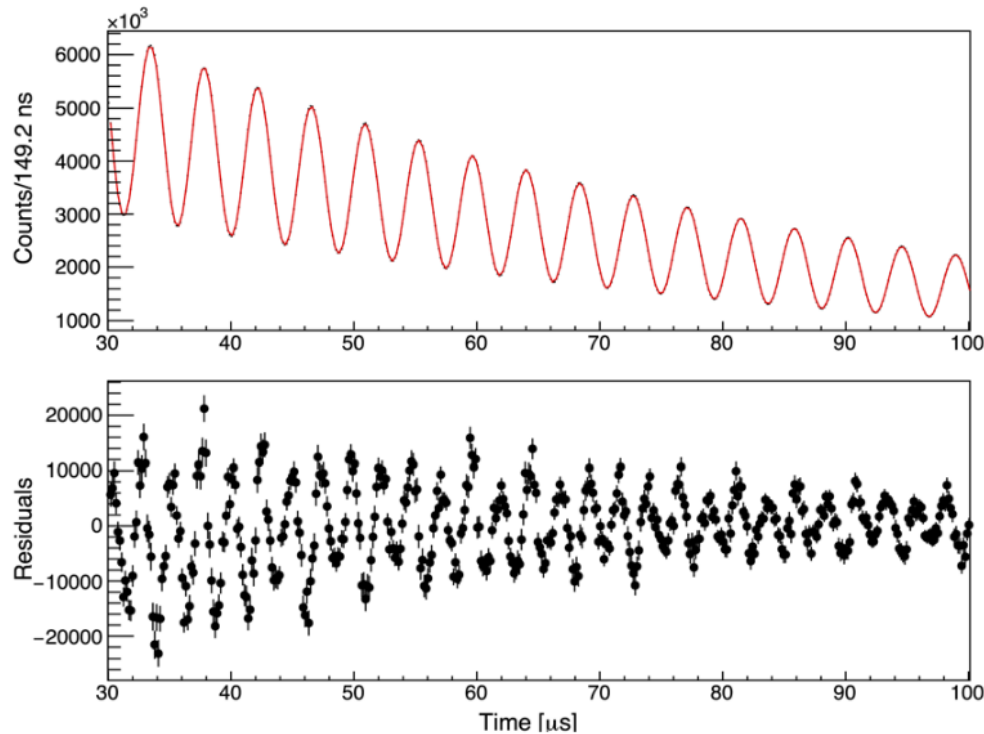
$$f(t) \propto N_0 e^{-\frac{t}{\gamma\tau}} [\langle N \rangle_{\text{thresh}} + \langle A \rangle_{\text{thresh}} \cos(\omega_a t - \langle \phi \rangle_{\text{thresh}})]$$



Account for complex beam dynamics
~27 free parameters in fit

Fitting the “wiggle” plot

$$f(t) \propto N_0 e^{-\frac{t}{\gamma\tau}} [\langle N \rangle_{\text{thresh}} + \langle A \rangle_{\text{thresh}} \cos(\omega_a t - \langle \phi \rangle_{\text{thresh}})]$$



Any time dependent phase shift will bias the frequency

Account for complex beam dynamics
~27 free parameters in fit

Magnetic field tracking

Magnetic field tracking

Trolley system

17 NMR probes

pulled through ring every ~3 days

measures spatial field dist. in storage region



Magnetic field tracking

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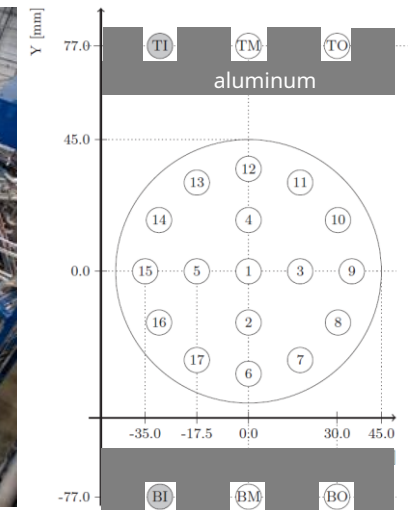


Fixed probe system

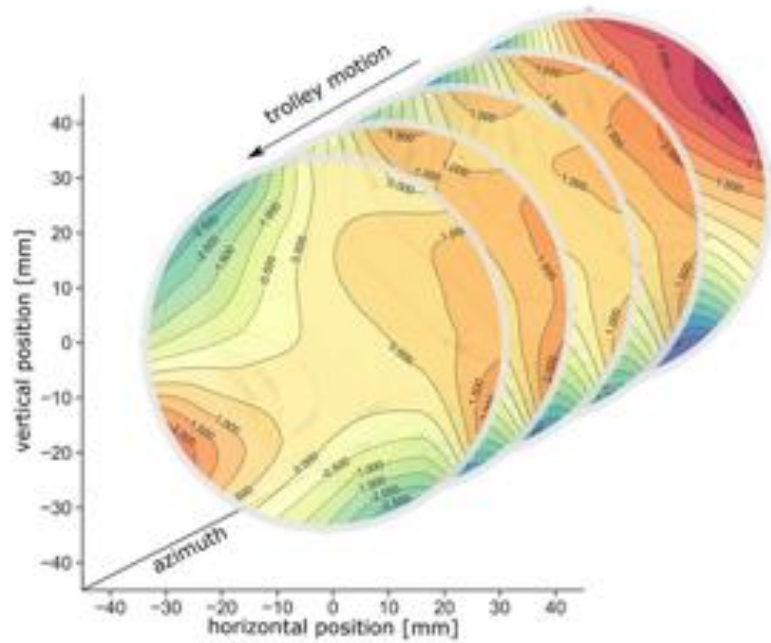
72 azimuthal location (stations)

tracks field drift 24/7

measures field differences (drift)

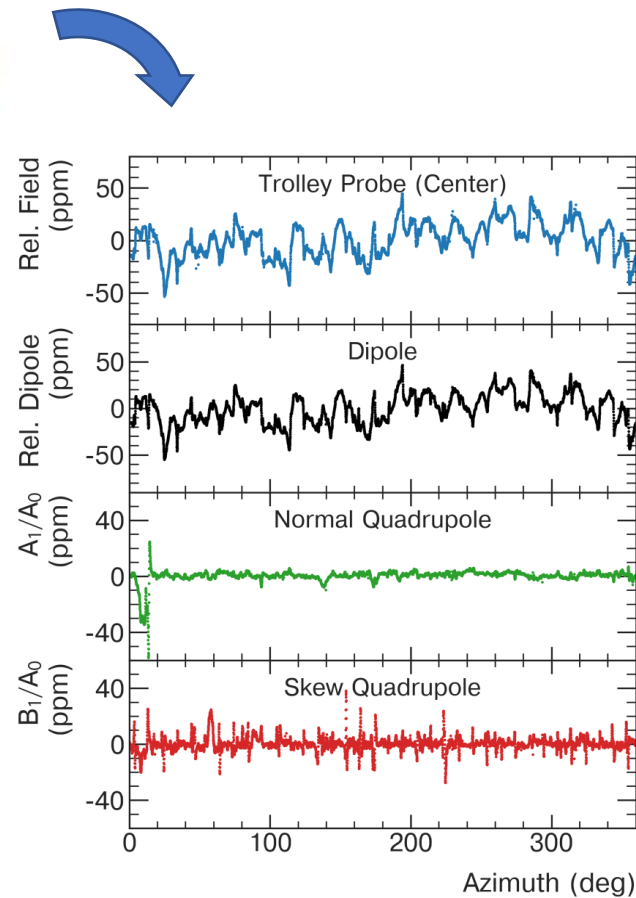
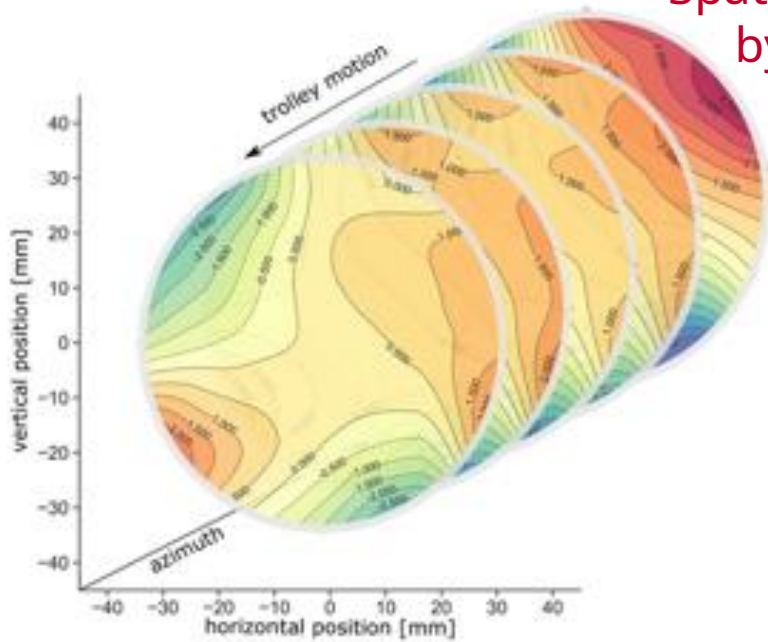


Magnetic field tracking



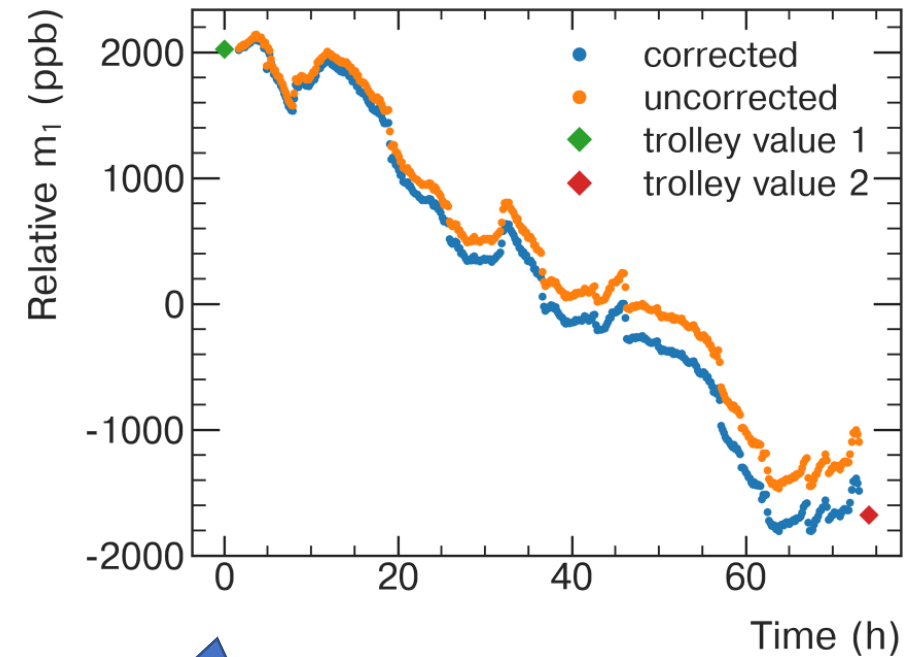
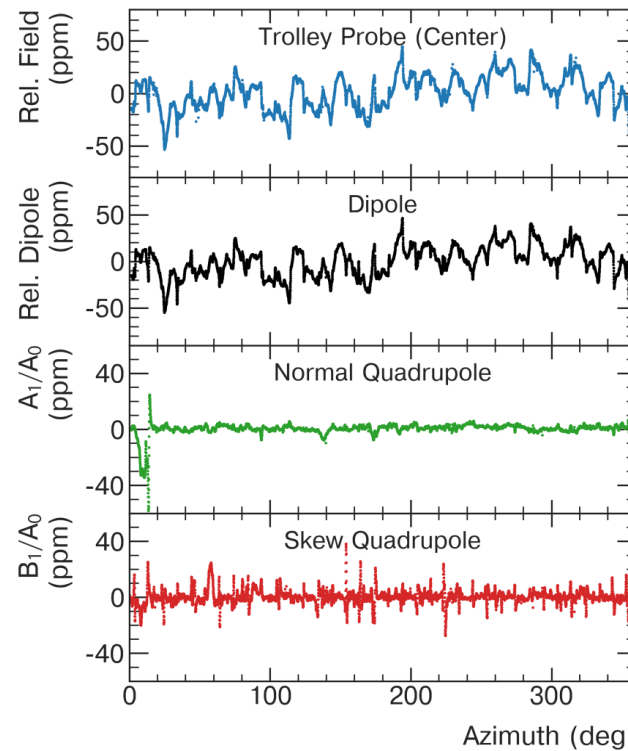
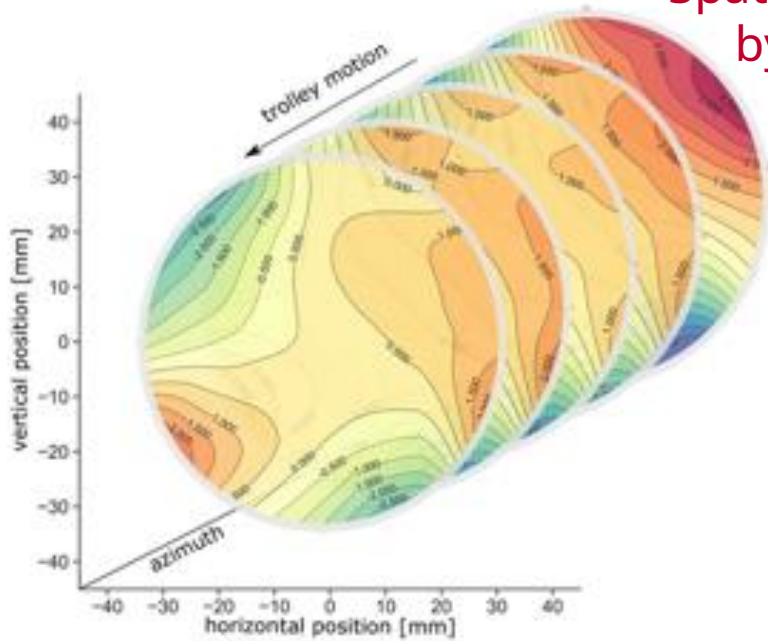
Magnetic field tracking

Spatial distribution described by multipole expansion



Magnetic field tracking

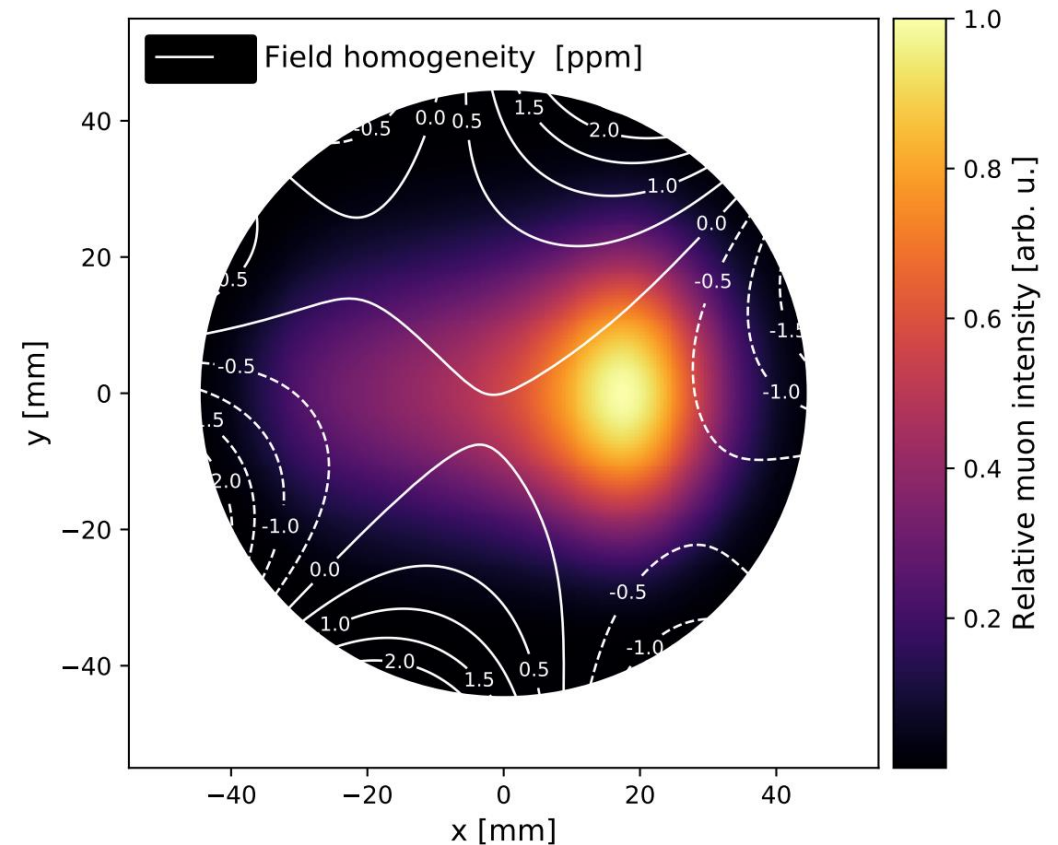
Spatial distribution described by multipole expansion



Muon weighted magnetic field

- We need the field seen by the muons
- Tracking magnetic field multipole moments
- Muon distribution given by tracker data and beam dynamics simulation

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}} \omega_a^{\text{meas}} (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{\text{calib}} \langle M(x, y, \phi) \omega'_p(x, y, \phi) \rangle (1 + B_k + B_q)}$$

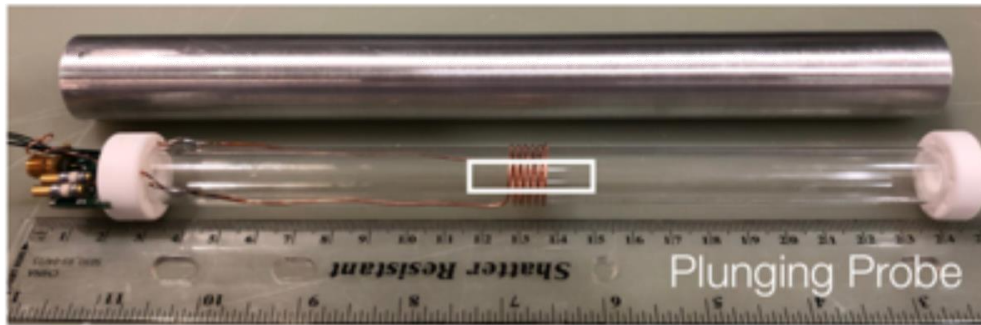


Field calibration

- Trolley is main device to measure the field
 - Trolley probes based on petroleum jelly
 - Needs calibration

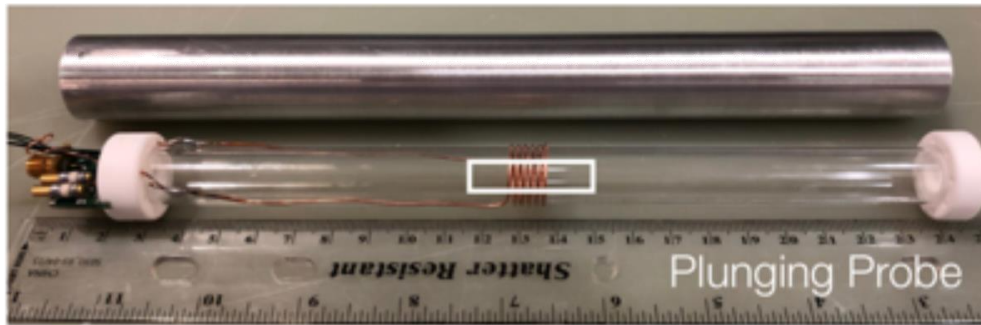
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- Absolute calibrated water probe
- Cross-calibrated at Argonne National Lab test magnet

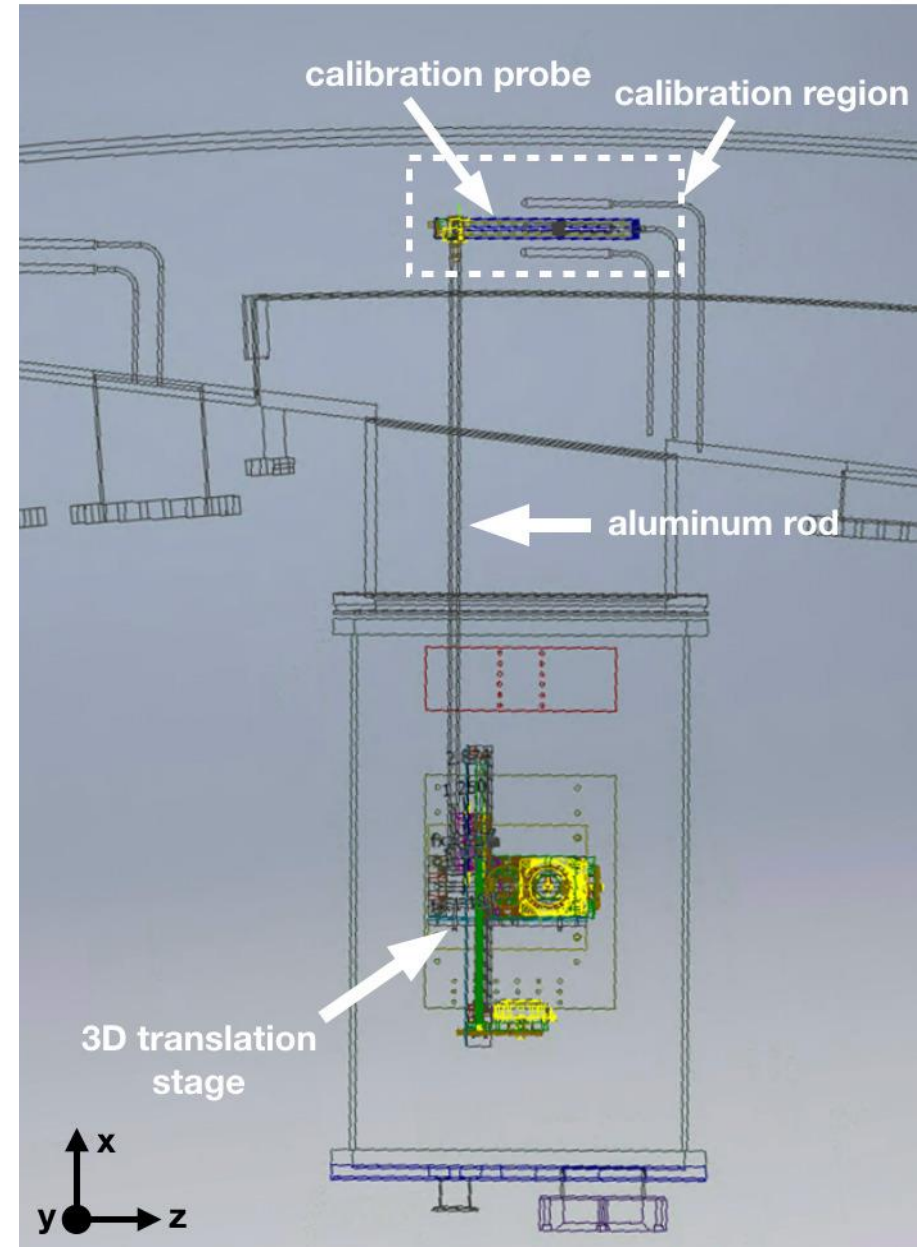


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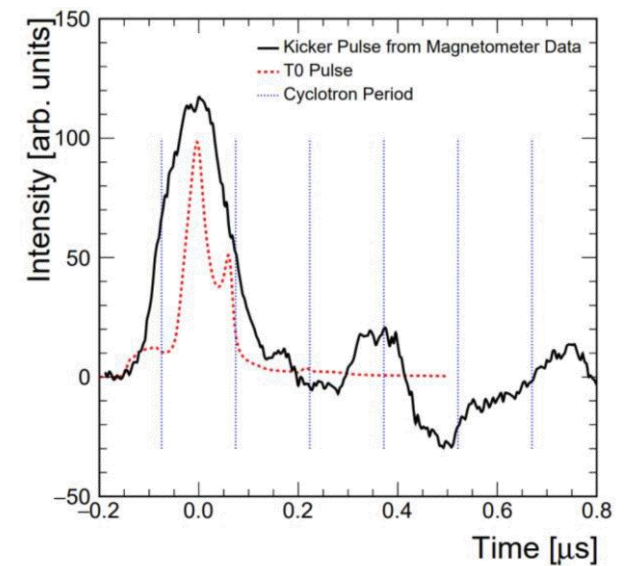
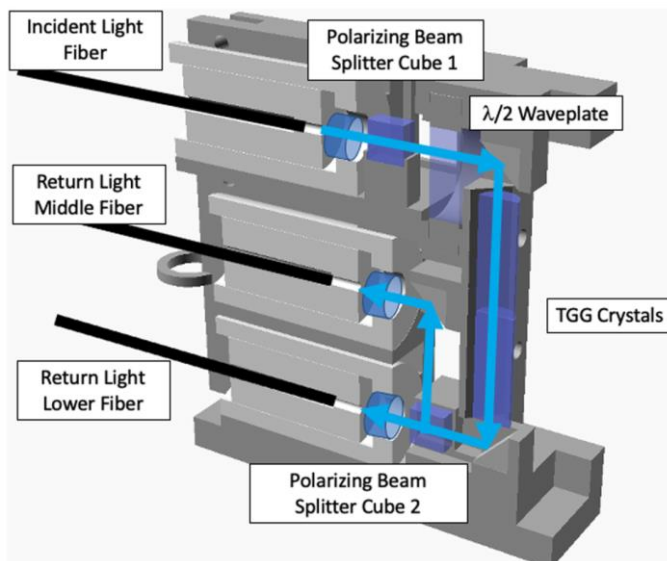


- Probe can be placed in ring by 3D translation stage
- Swap trolley and calibration probe ten times
- Derive calibration constants for each trolley probe



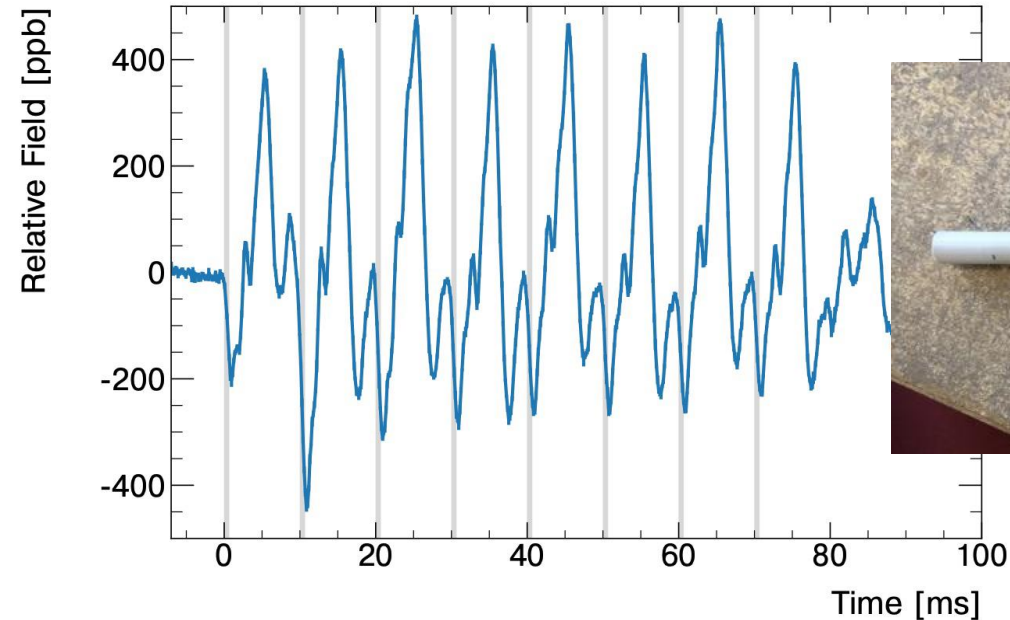
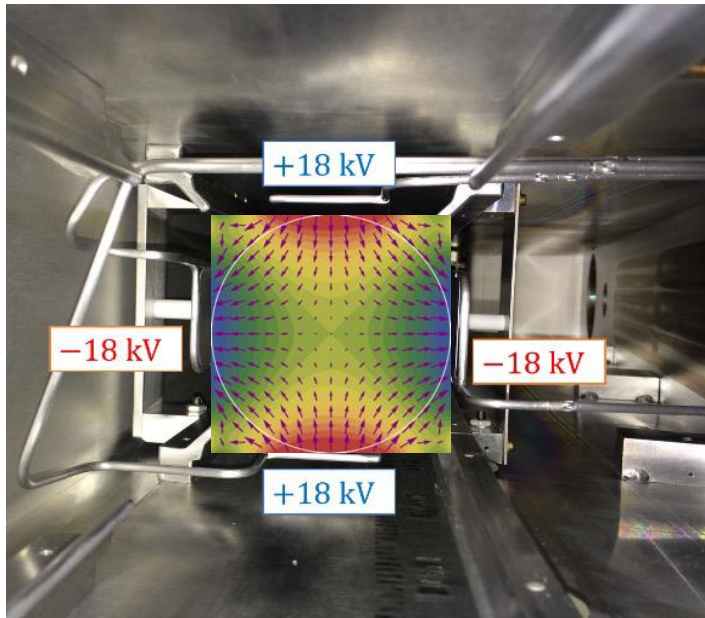
Kicker transient magnetic field

Kicker used to place beam on storage orbit
Kicker pulse induces 22mT field in radial direction
Measurement based on optical faraday rotation



Magnetic field quadrupole transients

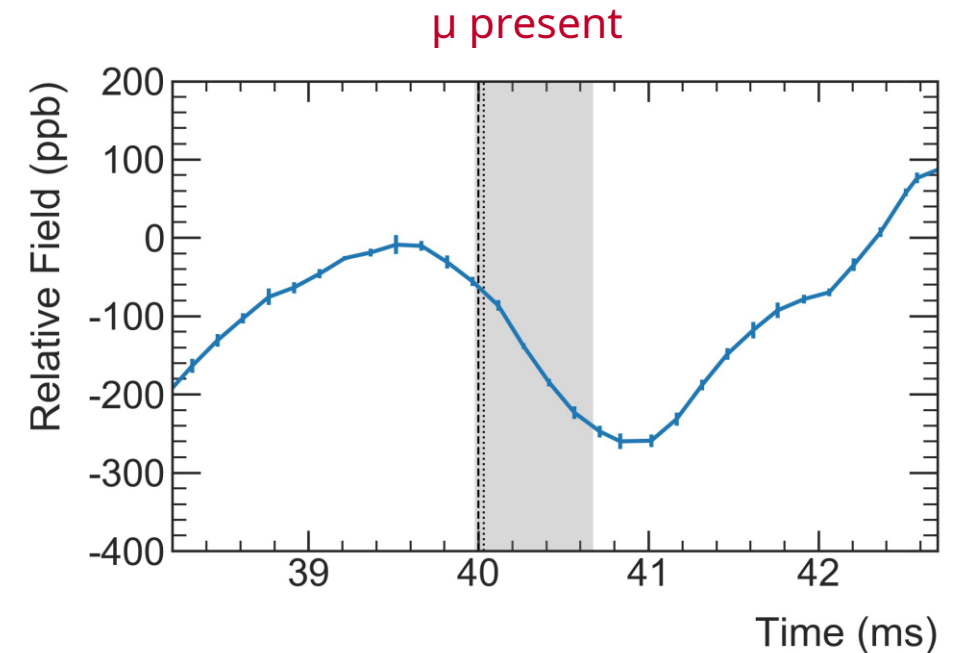
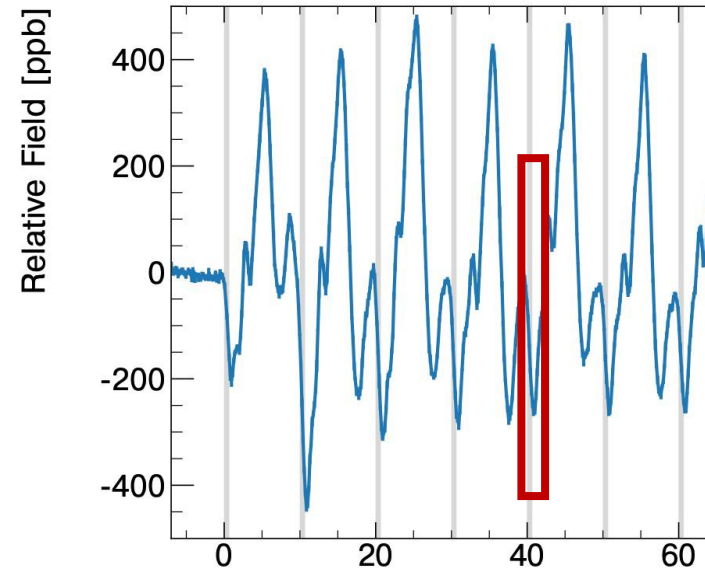
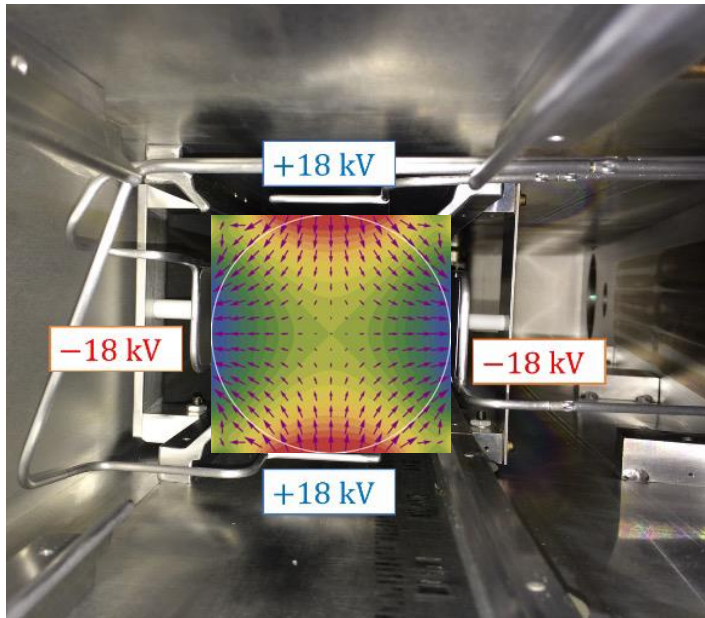
Pulsing electrostatic quadrupoles for beam confinement leads to magnetic field transient.



NMR probes run asynchronous with beam injection
Fast transient fields are shielded by aluminum in vacuum chambers

Magnetic field quadrupole transients

Pulsing electrostatic quadrupoles for beam confinement leads to magnetic field transient.



NMR probes run asynchronous with beam injection
Fast transient fields are shielded by aluminum in vacuum chambers

Uncertainties for Run 1

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}} \omega_a^{\text{meas}} (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{\text{calib}} \langle M(x, y, \phi) \omega'_p(x, y, \phi) \rangle (1 + B_k + B_q)}$$

Run 1

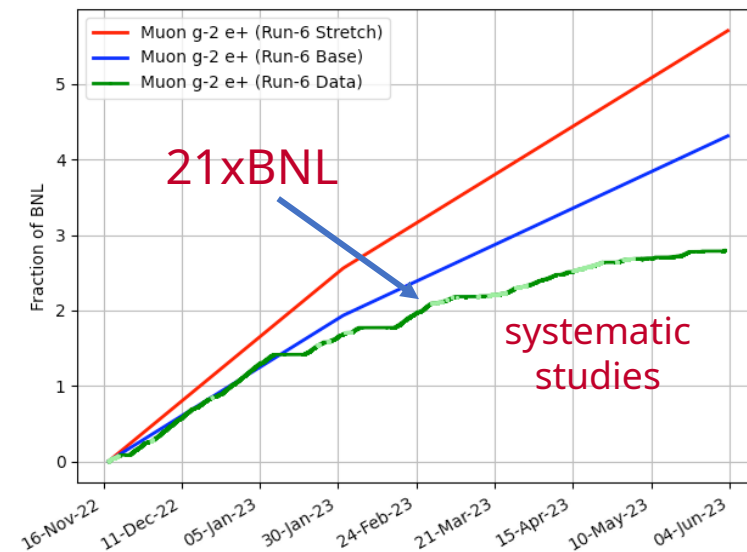
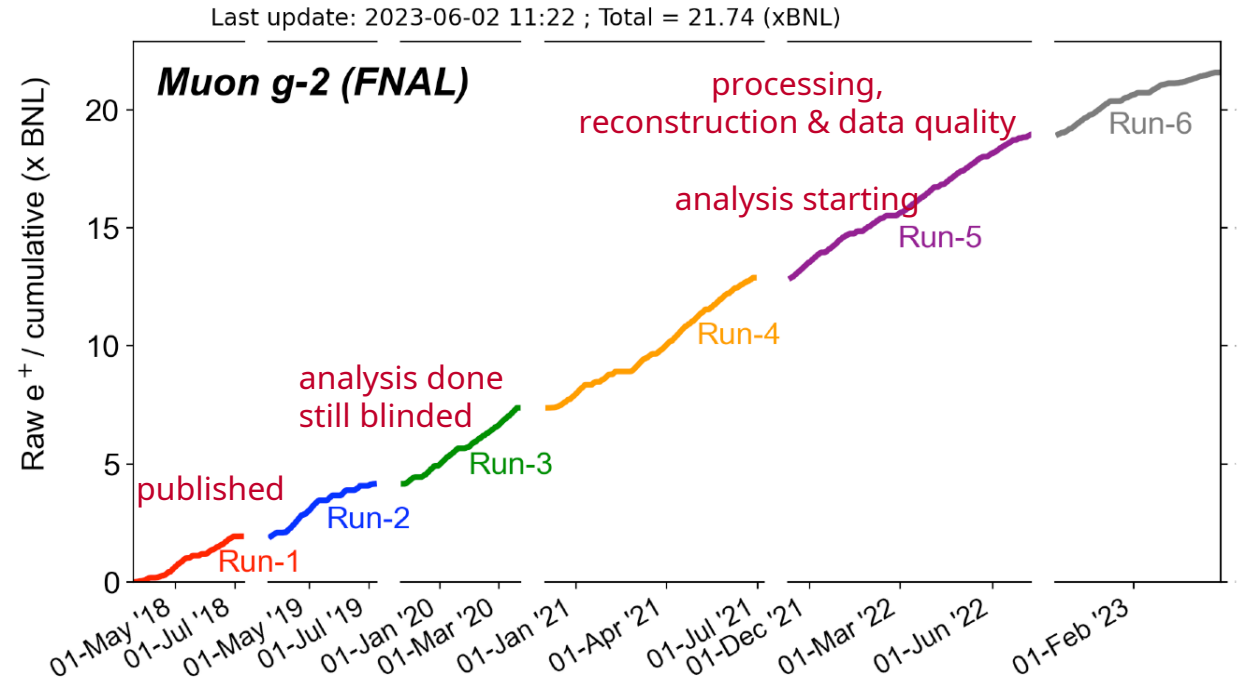
Design goal

Quantity	Correction Terms (ppb)	Uncertainty (ppb)	
ω_a^m (statistical)	–	434	100 ppb
ω_a^m (systematic)	–	56	
C_e	489	53	70 ppb
C_p	180	13	
C_{ml}	-11	5	
C_{pa}	-158	75	
$f_{\text{calib}} \langle \omega_p(x, y, \phi) \times M(x, y, \phi) \rangle$	–	56	70 ppb
B_k	-27	37	
B_q	-17	92	
$\mu'_p(34.7^\circ)/\mu_e$	–	10	
m_μ/m_e	–	22	
$g_e/2$	–	0	
Total systematic	–	157	100 ppb
Total fundamental factors	–	25	
Totals	544	462	140 ppb

- Improve statistics
→ take more data
- Systematics must be improved to achieve design goal
→ Reduce systematics in operations
- Improve understanding of systematic effects

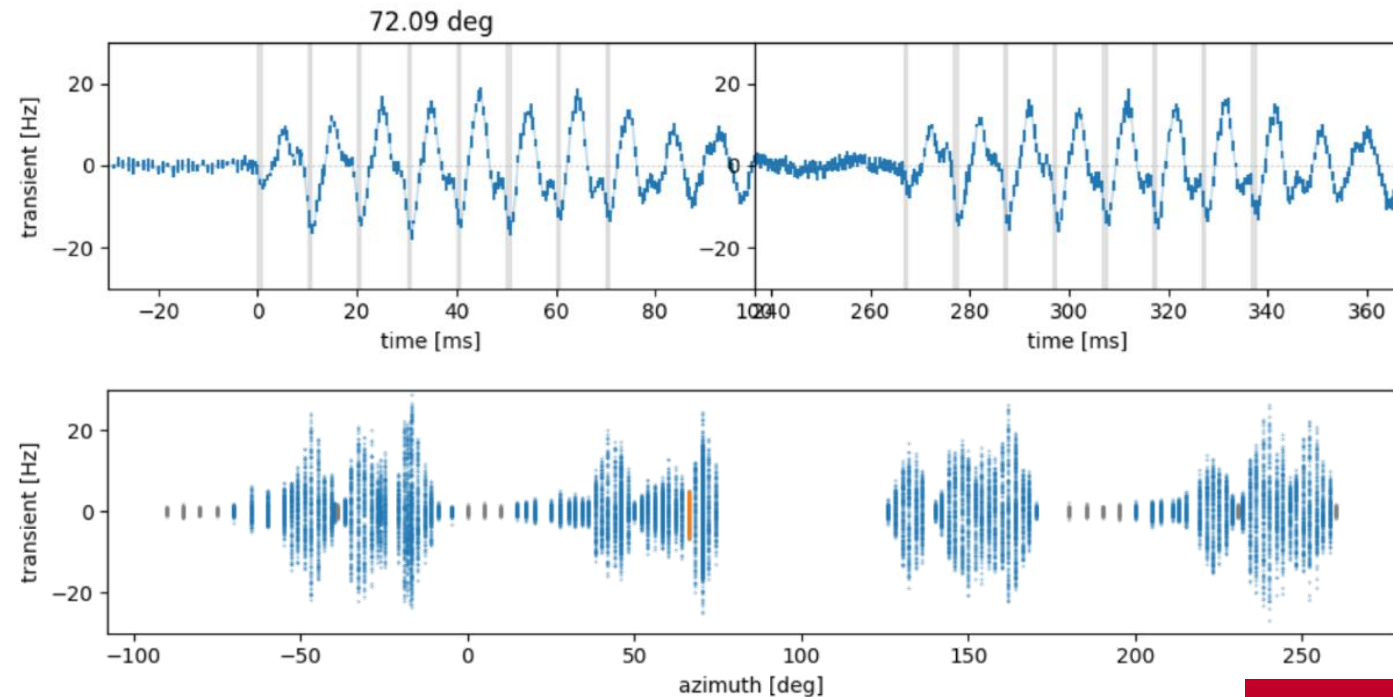
Data Taking

- Run 1 analysis published
 - Statistics \rightarrow ~ 462 ppb
 - Systematic \rightarrow 157 ppb
- Run2/3 analysis completed
 - Internal review, still blind
 - New result coming soon
 - Statistics \rightarrow ~ 230 ppb
 - Systematic \rightarrow ~ 110 ppb
- No negative muons
- Past design goal of 21 BNL /27.02
- Run6 focus on systematic studies
- Run 4/5/6 analysis starting



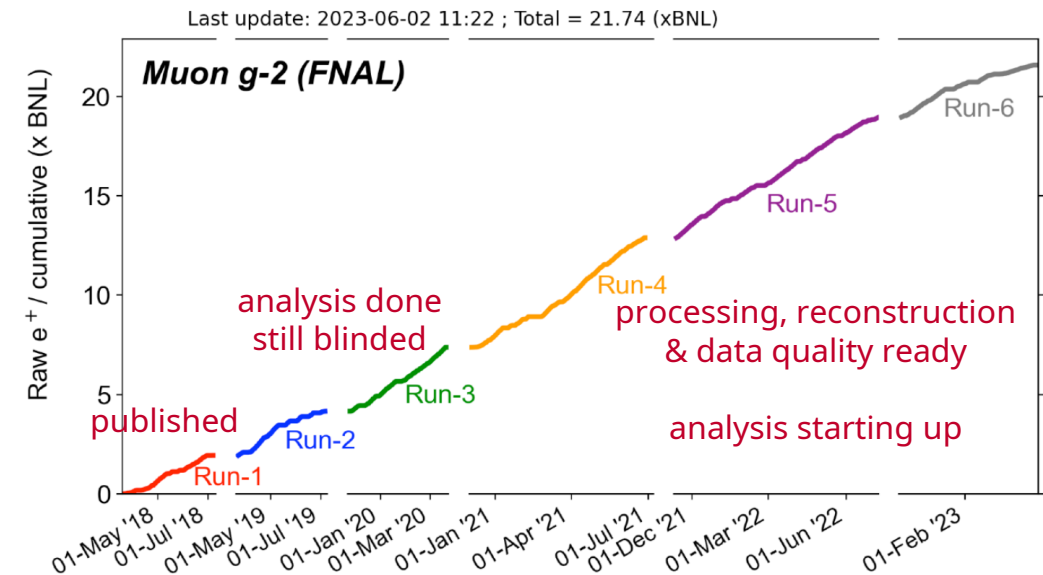
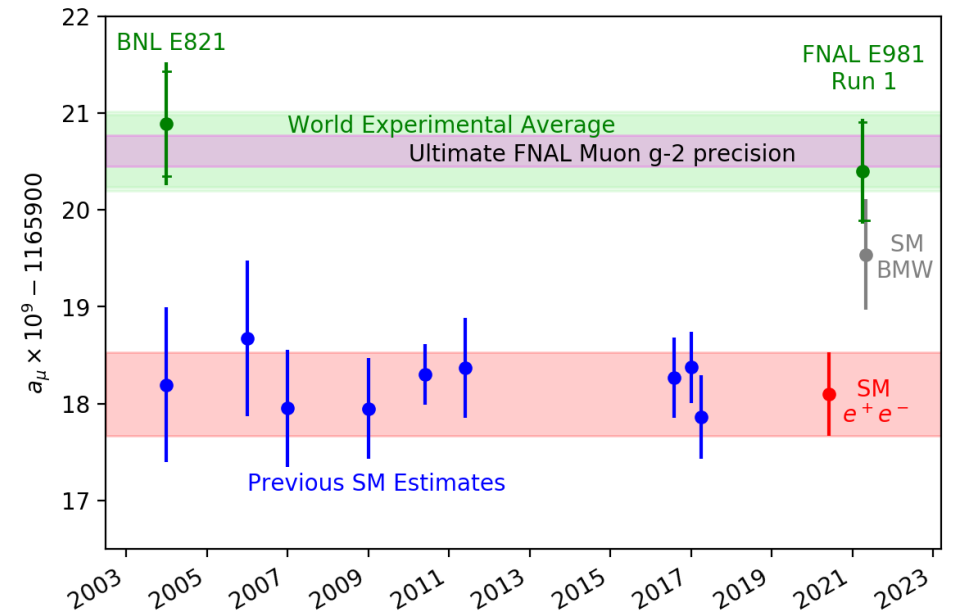
Improvements in Run 2/3

- ESQ transient dominating systematic in run1 (92ppb)
 - New effect (assume mechanical vibrations of quad plates)
 - Spatial & time structure unknown
 - Estimated from measurements in $\sim 20^\circ$ region
- Summer 2021 measurement campaign
 - Measure field with PEEK NMR probe synchronized to trigger
 - Scan delay between trigger & NMR measurement (20 min)
 - Measure spatial structure repeating at 92 azimuthal locations
 - Total uncertainty ~ 20 ppb



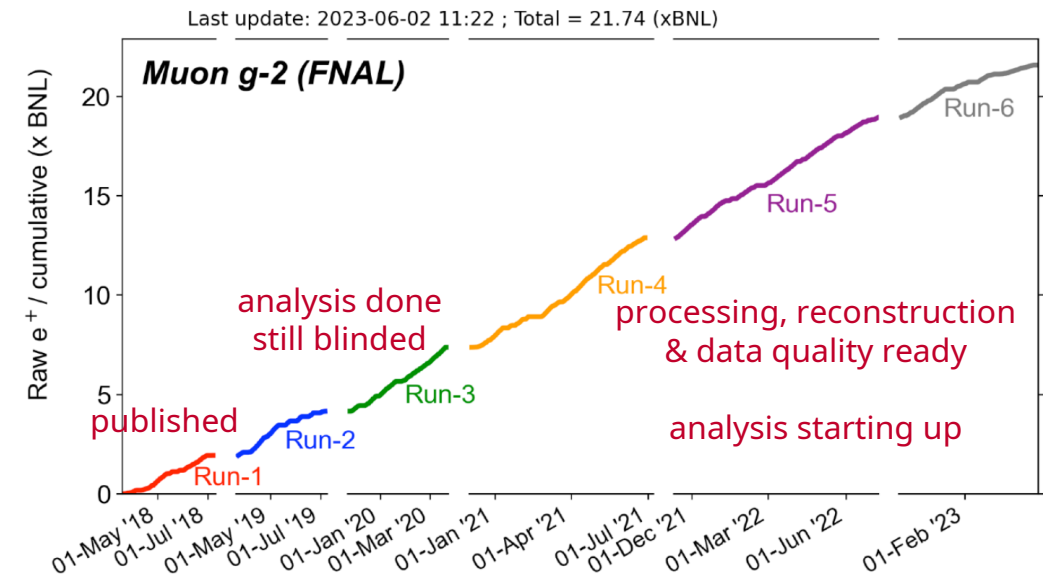
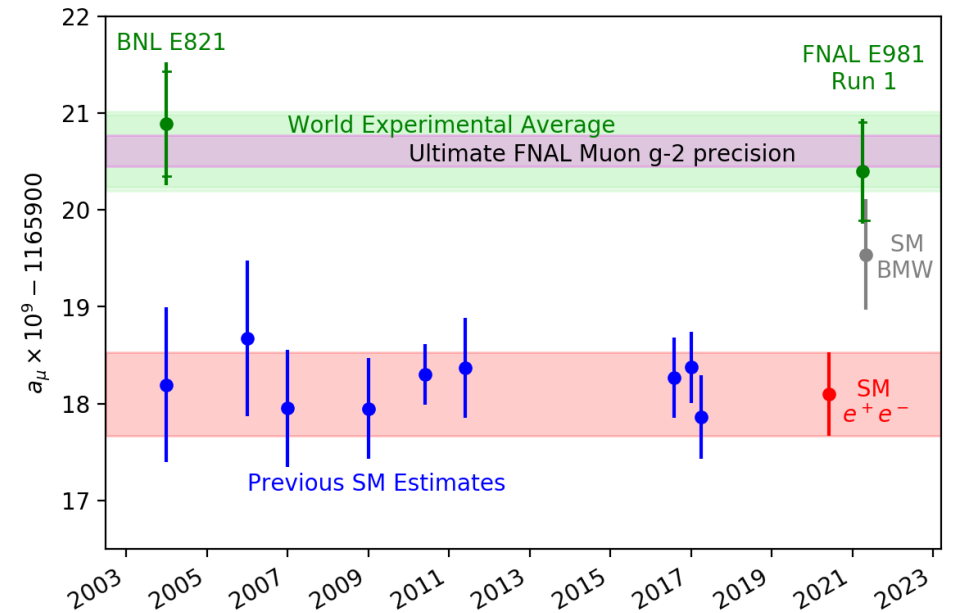
Summary

- High precision measurements of muon $g-2$ stringent test on SM theory
- First time a three-way comparison of a_μ is possible, very exciting
- Run 2 / 3 result soon
 - four times larger statistics
 - $\sim 1/3$ reduction in systematics
- Analysis for EDM on-going as well
- Reached 21 x BNL statistics and detailed systematic datasets
- Run 4/5/6 analysis ramping up



Summary

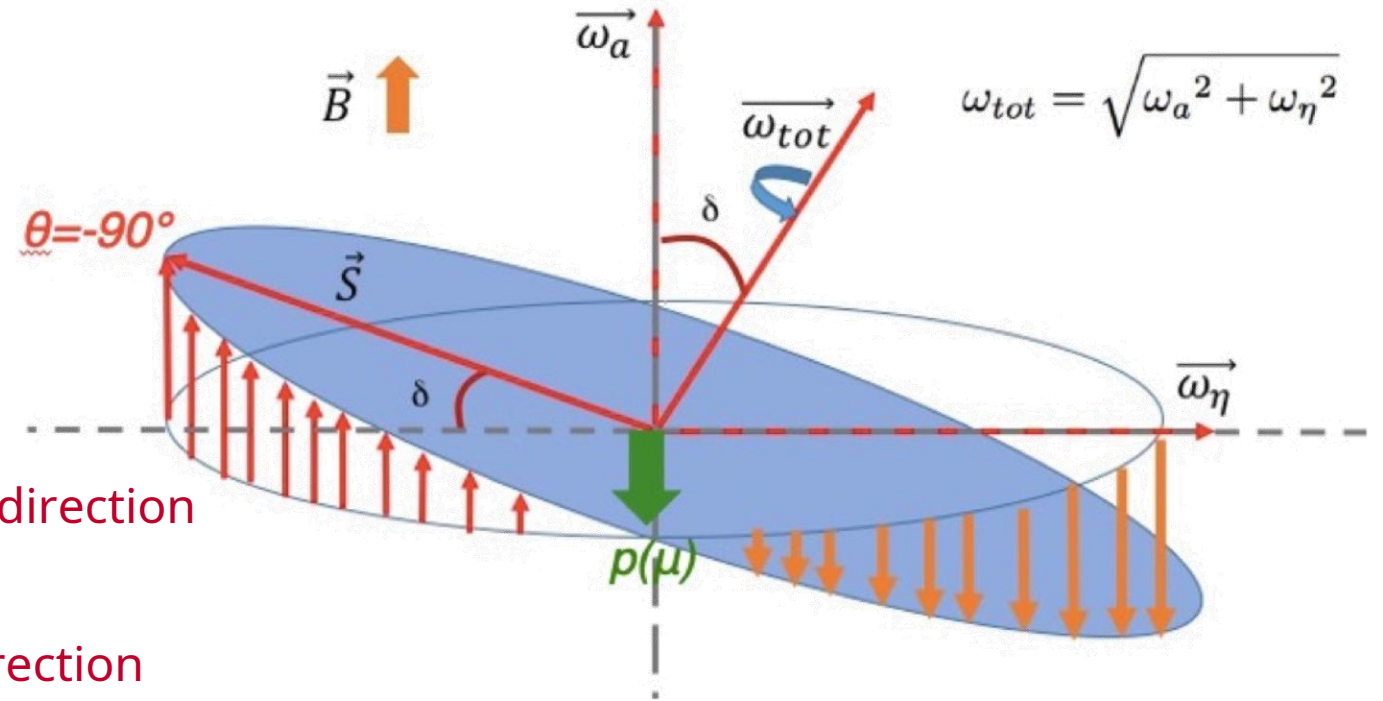
- High precision measurements of muon $g-2$ stringent test on SM theory
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Backup

EDM with E989

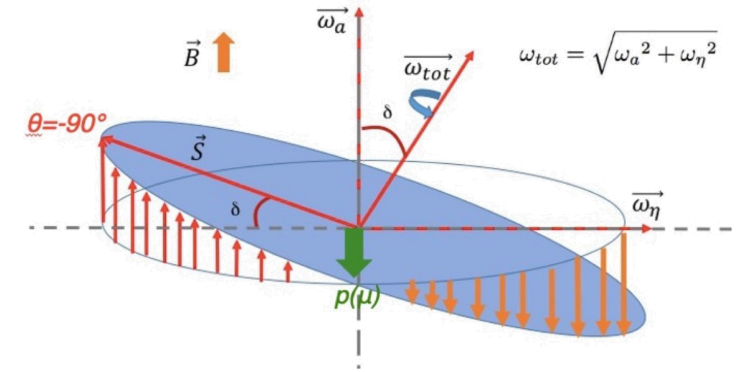
- Muon EDM leads to spin precession as well
- MDM leads to rotation around vertical direction
- EDM leads to rotation around radial direction



$$\vec{\omega}_s = \underbrace{-\frac{q}{m} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]}_{\text{MDM}} + \underbrace{\frac{\eta q}{2mc} \left[\vec{E} + c\vec{\beta} \times \vec{B} \right]}_{\text{EDM}}$$

Rotation around vertical
Rotation around radial

EDM with E989



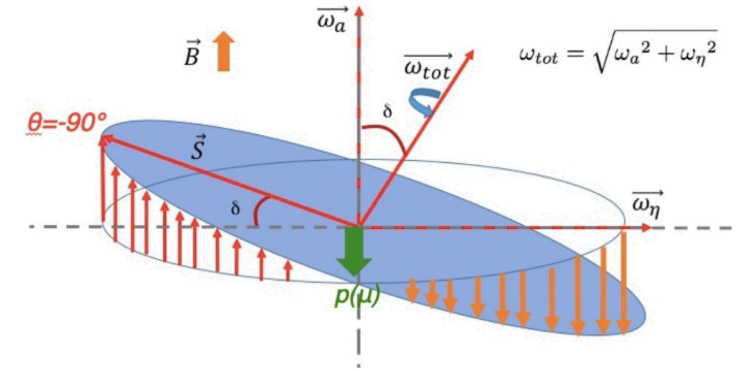
1. Deviation in measured $|\omega_a|$

- Assuming $a_\mu^{\text{Exp}} - a_\mu^{\text{SM}}$ purely caused by EDM results in $d_\mu = (2.3 \pm 0.3) \times 10^{-19} e \cdot \text{cm}$
- Exceeds current upper limit of $|d_\mu| < 1.8 \times 10^{-19} e \cdot \text{cm}$ at 95% C.L.

$$\vec{\omega}_s = \underbrace{-\frac{q}{m} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]}_{\text{MDM}} + \underbrace{\frac{\eta q}{2mc} \left[\vec{E} + c\vec{\beta} \times \vec{B} \right]}_{\text{EDM}}$$

Rotation around vertical
Rotation around radial

EDM with E989



2. Observe vertical decay angle

- Measure positron momentum vector with tracker
- Determine vertical decay angle $\theta_y = \arcsin(p_y/p)$
- EDM signal oscillates with 180° out-of-phase w.r.t ω_a

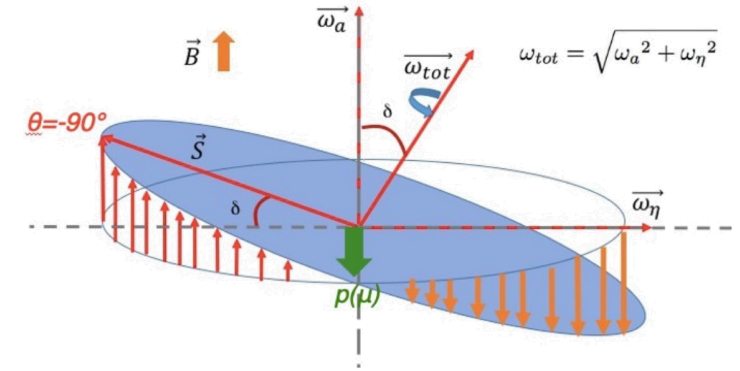
$$\langle \theta_y \rangle(t) = A_{g-2} \cos(\omega_a t + \phi) + A_{\text{EDM}} \sin(\omega_a t + \phi) + c$$

- Systematics: tracker acceptance, tracker alignment, radial magnetic field
- Statistical limited in BNL measurement
- Run 1 analysis nearly ready but still blinded
- Limit if $A_{\text{EDM}}=0$ $|d_\mu| < 2.0 \times 10^{-19} e \cdot \text{cm}$

$$\vec{\omega}_s = \underbrace{-\frac{q}{m} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]}_{\text{MDM}} + \underbrace{\frac{\eta q}{2mc} \left[\vec{E} + c\vec{\beta} \times \vec{B} \right]}_{\text{EDM}}$$

Rotation around vertical
Rotation around radial

EDM with E989



3. Frozen spin Observe vertical decay angle

- For muon g-2 we operate at magic momentum
- Idea: apply radial dipolar electric field to vanish MDM contribution
- Operate E989 with ~ 300 MeV/c muons, $B \sim 0.13$ T and $E_r \sim 0.77$ MV/m
- No spin rotation in vertical direction (frozen spin)
- Tested beam line to deliver ~ 300 MeV/c muons
- Simulations on-going to investigate measurement principle

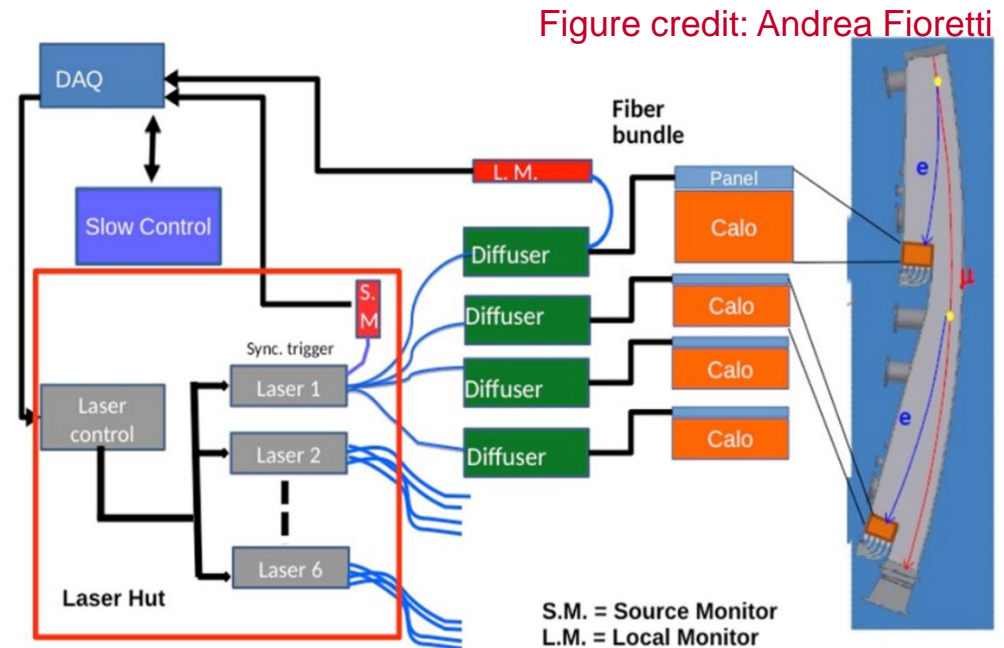
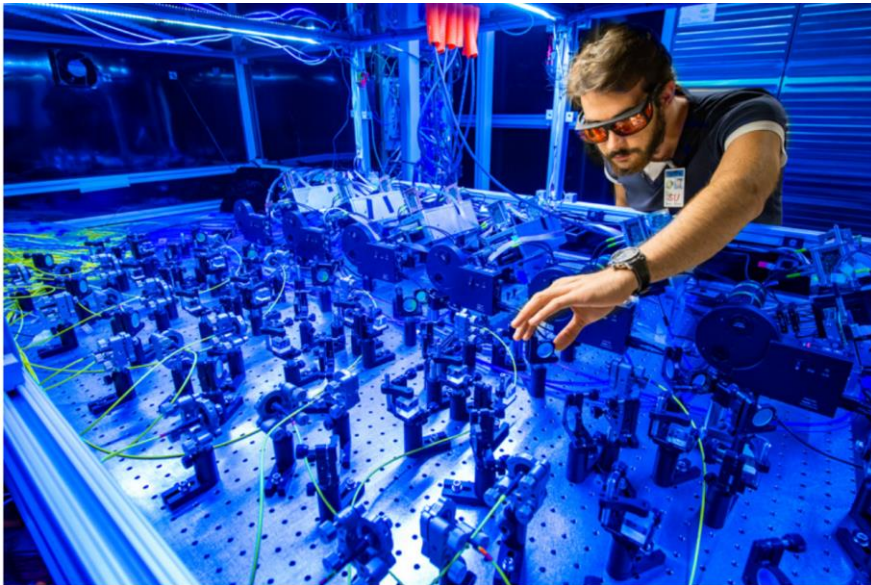
$$\vec{\omega}_s = \underbrace{-\frac{q}{m} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]}_{\text{MDM}} + \underbrace{\frac{\eta q}{2mc} \left[\vec{E} + c\vec{\beta} \times \vec{B} \right]}_{\text{EDM}}$$

=0
Rotation around vertical
Rotation around radial

View inside the vacuum chambers



Laser calibration system



Inject laser pulses systematically also during beam operation (about 10% of time)

Long term gain changes due to temperature changes

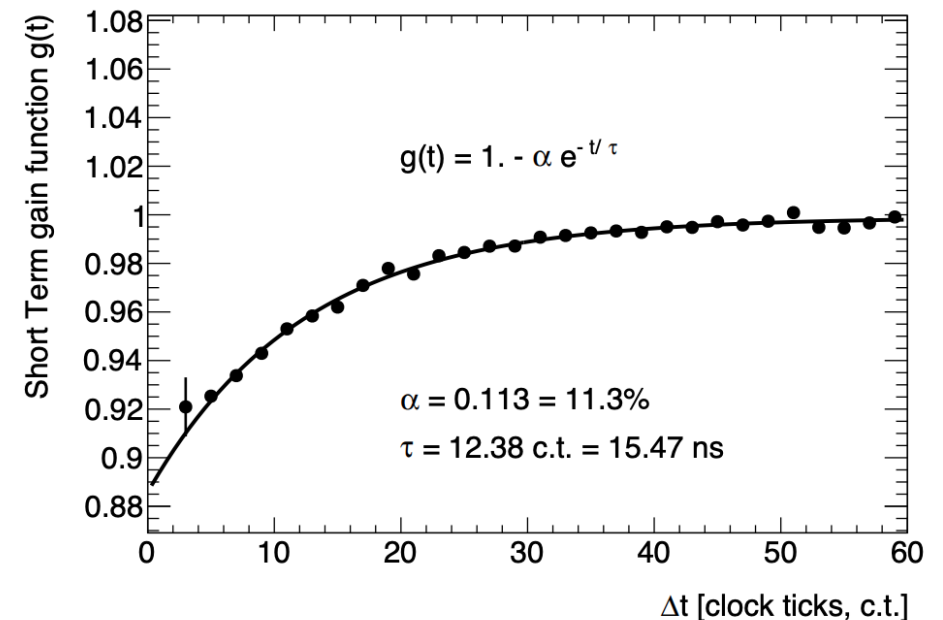
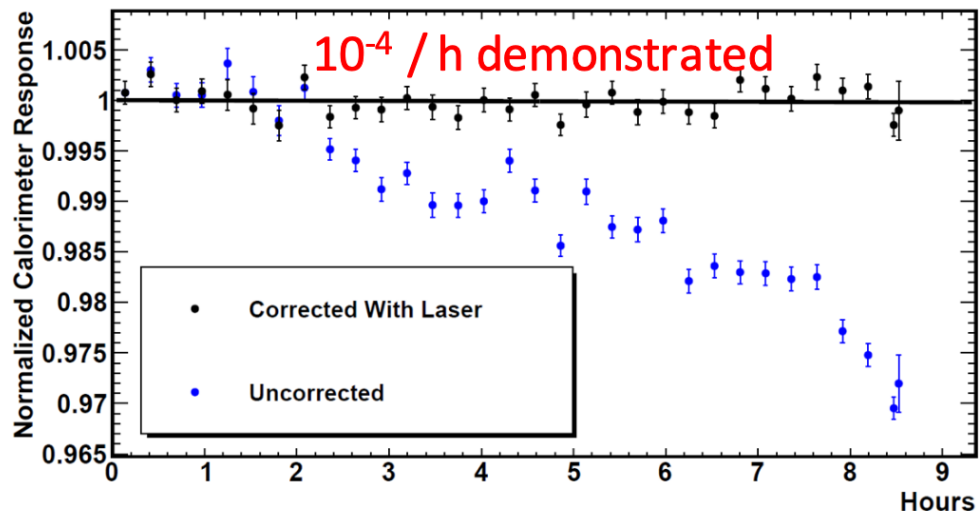
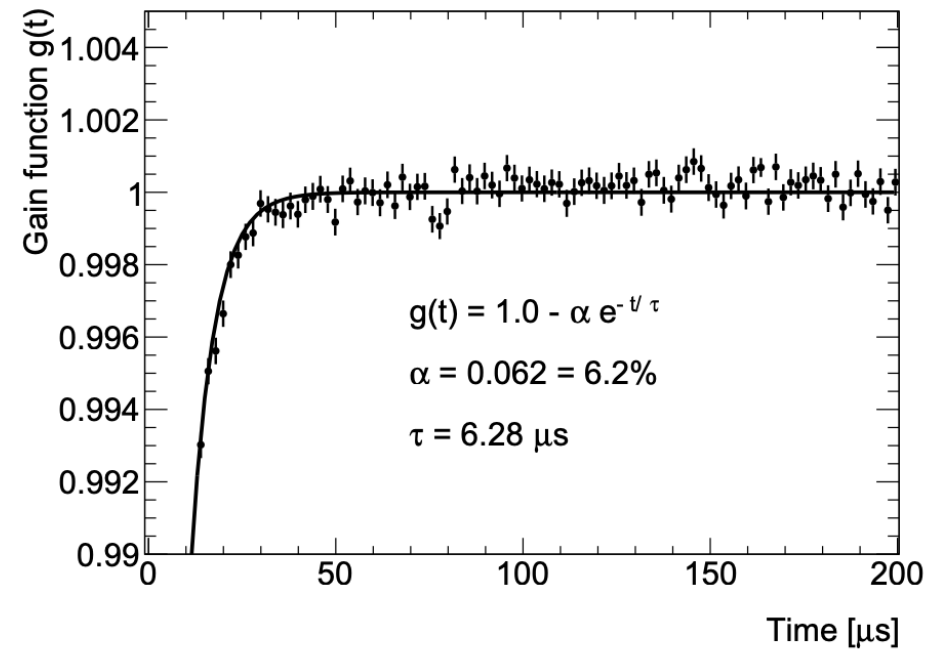
Short term gain drops from initial beam flash at injection or consecutive hits

Gain Stability

Long term gain changes due to temperature changes
Long term gain changes can be corrected

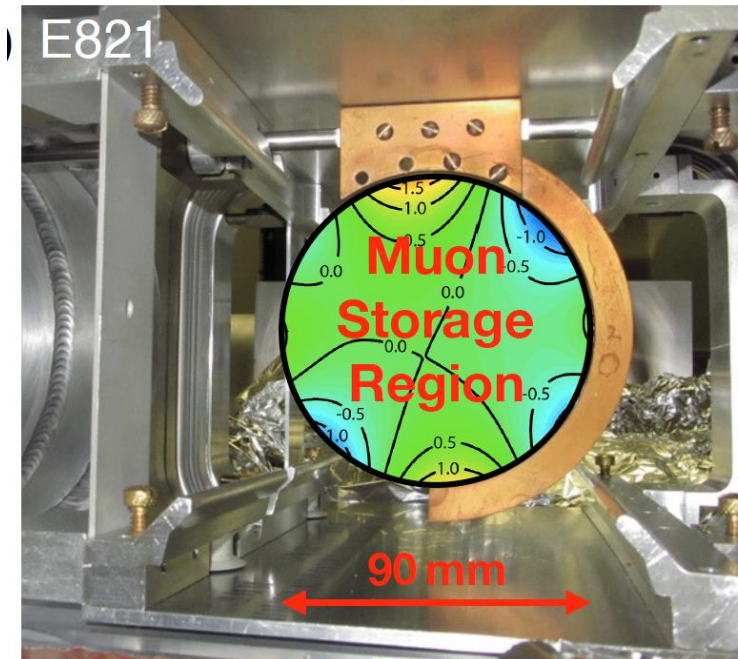
Short term gain drops

- Initial beam flash at injection
- Consecutive hits



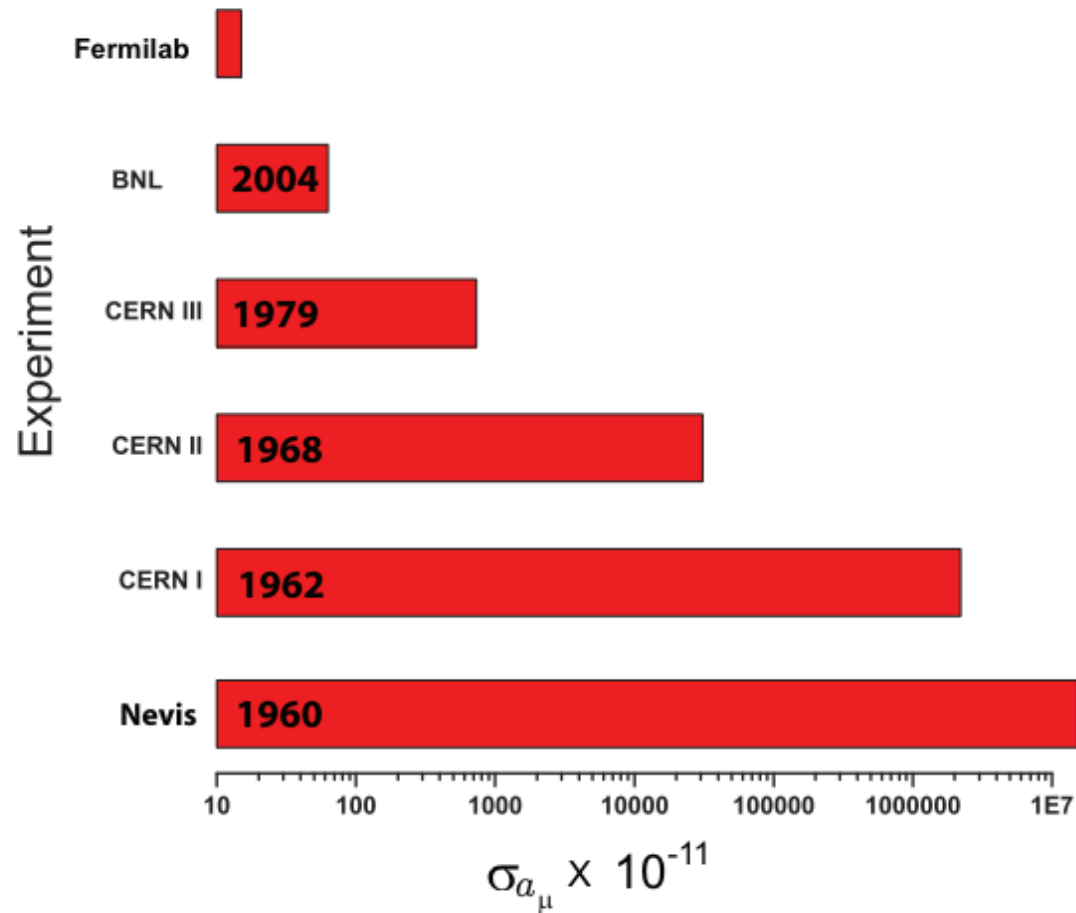
Scaping of beam edges

- Beam dynamics could make muons oscillated into physical objects around the muon storage area
- potential early-to-late in muon loss factor
- First apply small vertical focusing
- Edge of stored muons collide with collimators
- Second apply higher vertical focusing
- Stored muons well separated from collimators



Technique developed over 40 years

Goal: 100ppb statistical \oplus 100ppb systematic uncertainty



Superconducting storage ring magnet
Muon Injection and magnetic kicker
Superconducting inflector magnet

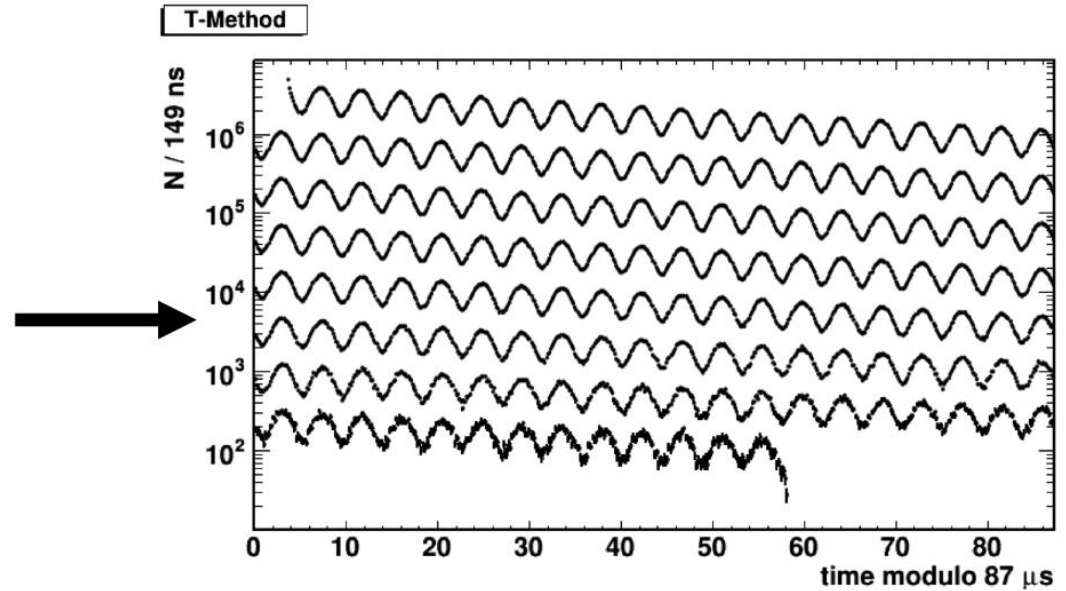
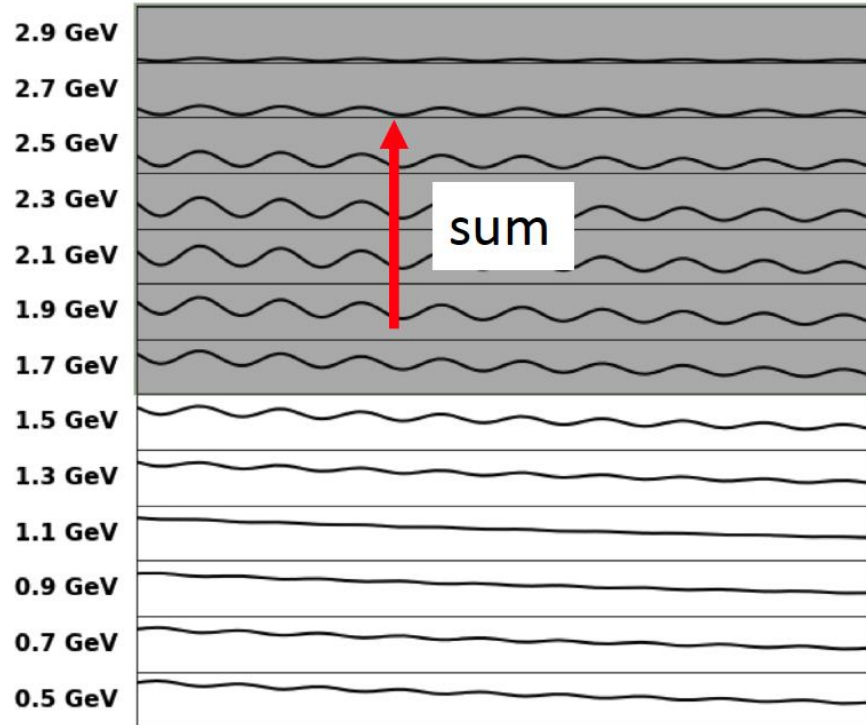
NMR technique

Magic momentum technique

Storage ring technique to measure $g-2$

Measured g_μ from muon at rest

The “wiggle” plot



$$N(t) = N_0(E) e^{-\frac{t}{\gamma\tau}} [1 + A(E) \cos(\omega_a t - \phi(E))]$$

Exponential decay from muon lifetime modulated with $\omega_a = a_\mu \frac{e}{m_\mu} B$

Beam dynamics corrections

- Electric field correction
 - Finite momentum distribution, not all at magic momentum
 - Debunching
 - determine momentum distribution
 - determine equilibrium radius distribution
- Pitch correction
 - Vertical momentum component of muons
 - Trackers measure vertical oscillation amplitude
- Muon loss correction
 - Lose muons due to collisions instead of decay
 - Different for low and high energy muons, early-to-late effect
- Phase difference depending on decay position
 - Corrected from Beam dynamic simulation

$$C_e = -2n(1 - n)\beta^2 \frac{\langle x_e^2 \rangle}{R_0^2}$$

$$C_p = \frac{n}{4R_0^2} \langle A^2 \rangle$$

$$C_{ml}$$

$$C_{pa}$$

22 parameter fit

$$N_0 e^{-\frac{t}{\tau}} (1 + A \cdot A_{BO}(t) \cos(\omega_a t + \phi \cdot \phi_{BO}(t))) \cdot N_{CBO}(t) \cdot N_{VW}(t) \cdot N_y(t) \cdot N_{2CBO}(t) \cdot J(t)$$

$$A_{BO}(t) = 1 + A_A \cos(\omega_{CBO}(t) + \phi_A) e^{-\frac{t}{\tau_{CBO}}}$$

$$\phi_{BO}(t) = 1 + A_\phi \cos(\omega_{CBO}(t) + \phi_\phi) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{CBO}(t) = 1 + A_{CBO} \cos(\omega_{CBO}(t) + \phi_{CBO}) e^{-\frac{t}{\tau_{CBO}}}$$

$$N_{2CBO}(t) = 1 + A_{2CBO} \cos(2\omega_{CBO}(t) + \phi_{2CBO}) e^{-\frac{t}{2\tau_{CBO}}}$$

$$N_{VW}(t) = 1 + A_{VW} \cos(\omega_{VW}(t)t + \phi_{VW}) e^{-\frac{t}{\tau_{VW}}}$$

$$N_y(t) = 1 + A_y \cos(\omega_y(t)t + \phi_y) e^{-\frac{t}{\tau_y}}$$

$$J(t) = 1 - k_{LM} \int_{t_0}^t \Lambda(t) dt$$

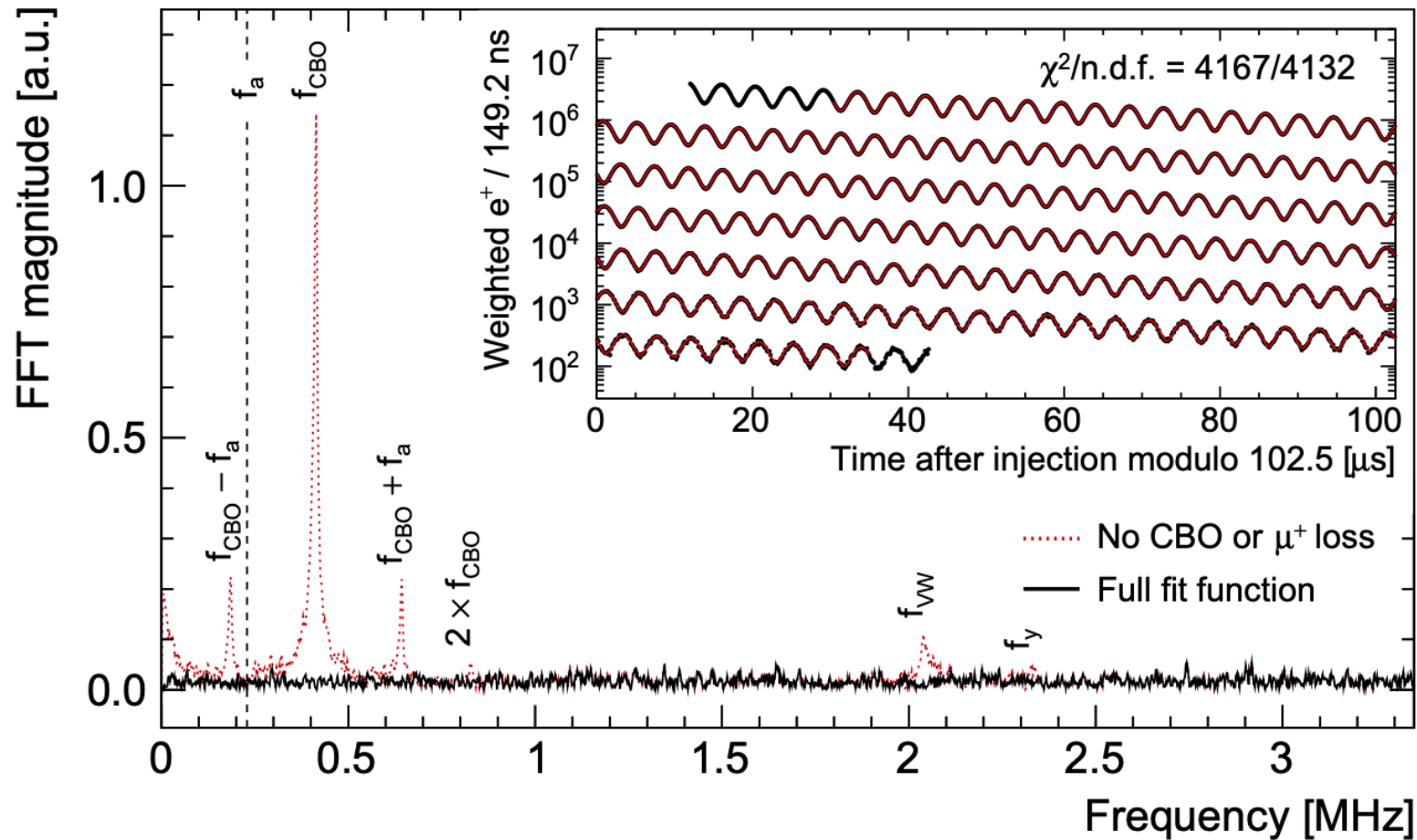
$$\omega_{CBO}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

$$\omega_y(t) = F \omega_{CBO}(t) \sqrt{2\omega_c / F \omega_{CBO}(t) - 1}$$

$$\omega_{VW}(t) = \omega_c - 2\omega_y(t)$$

Beam dynamics effects have to be considered

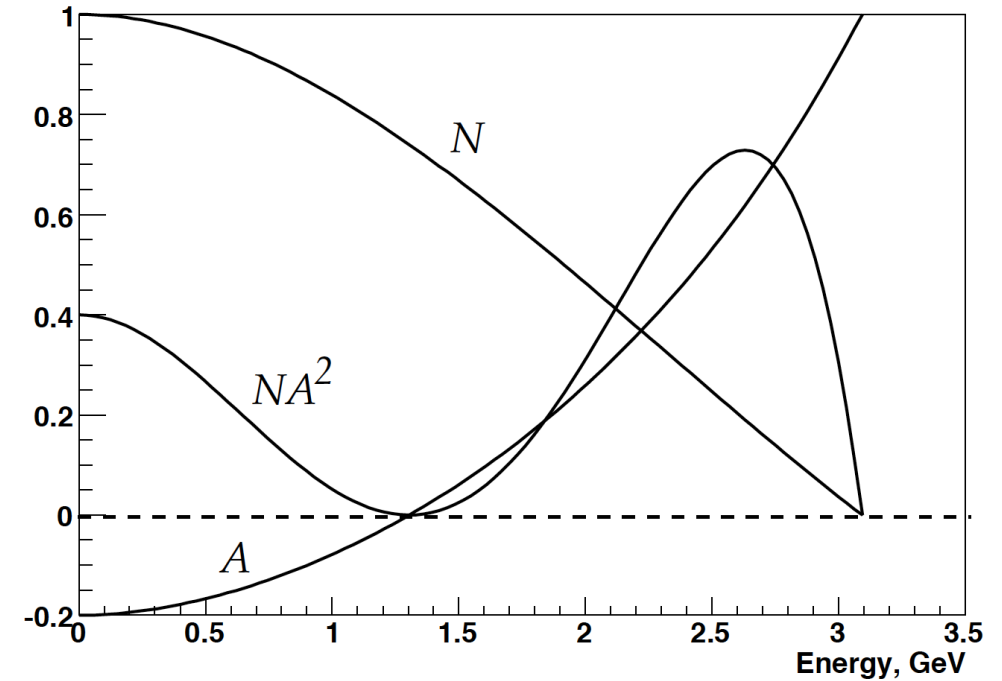
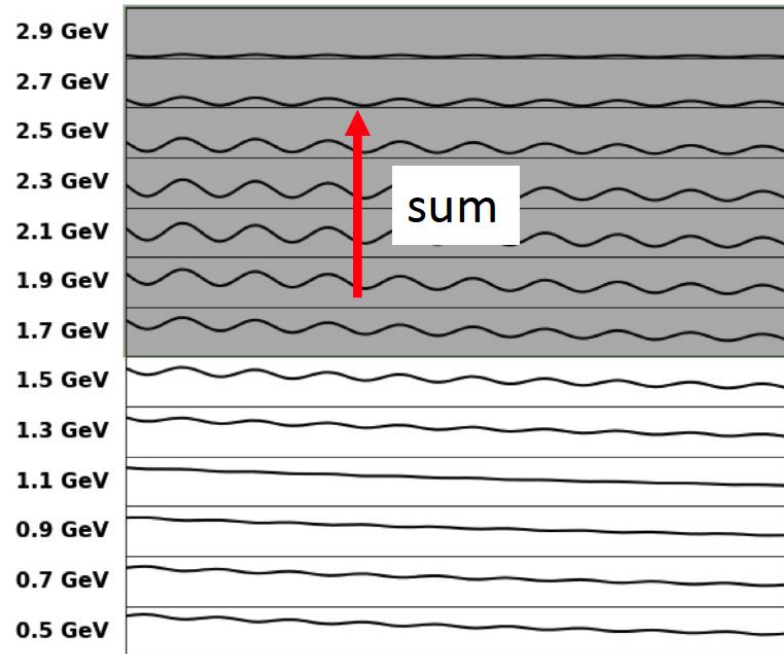
22 parameter fit



$$t) \cdot N_{2\text{CBO}}(t) \cdot J(t)$$

Beam dynamics effects have to be considered

Asymmetry weighted method



- Asymmetry is energy dependent
- High energy positrons have stronger asymmetry
- Introduce weight proportional to asymmetry

Ratio method

- Split positrons randomly in four sets
- Time shift one set by $+T_a/2$ and one by $-T_a/2$

- Build the ratio

$$r(t) = \frac{[u_+(t) - v_1(t)] + [u_-(t) - v_2(t)]}{[u_+(t) + v_1(t)] + [u_-(t) + v_2(t)]}$$

- Gets rid of exponential decay and any slow drift

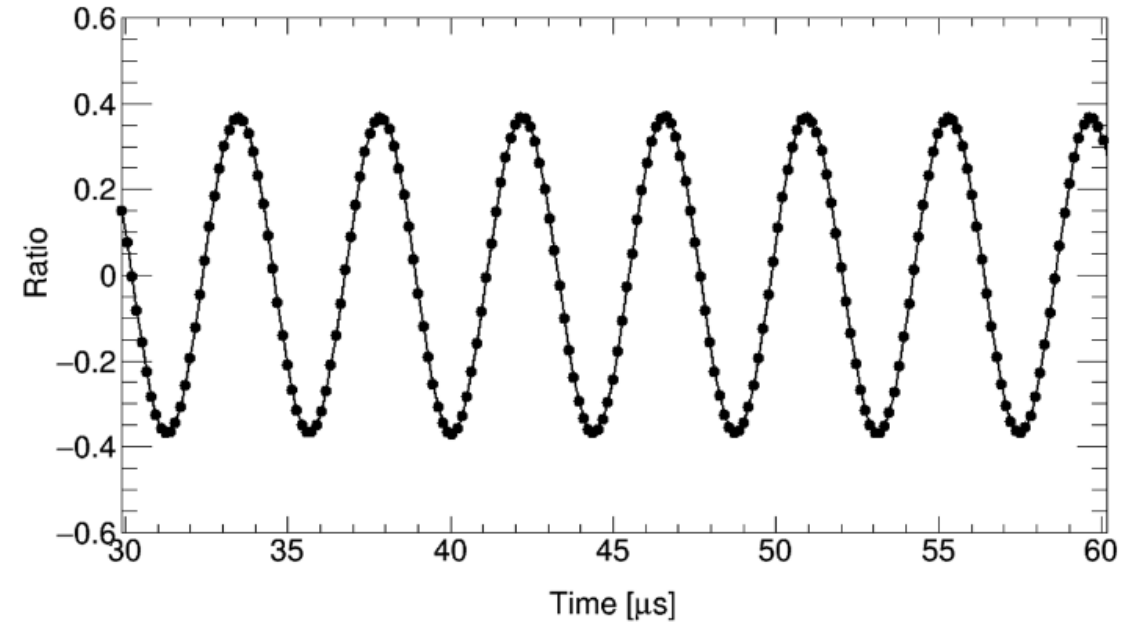
$$r(t) = A \cos(\omega_a^m t + \phi) - \frac{1}{16} \left(\frac{T_a}{\gamma \tau_\mu} \right)^2 + \mathcal{O}((T_a / (4\gamma \tau_\mu))^4)$$

$$u_+(t) = \frac{1}{4} n(t + T_a/2),$$

$$u_-(t) = \frac{1}{4} n(t - T_a/2),$$

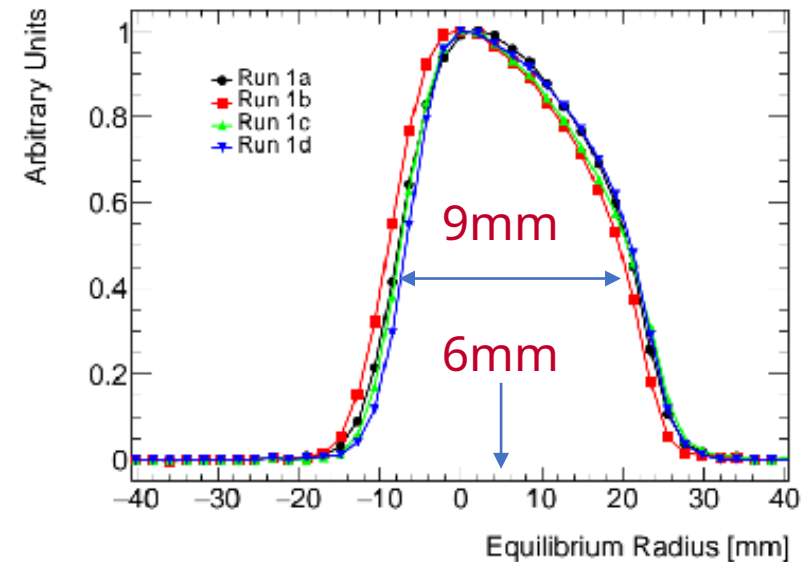
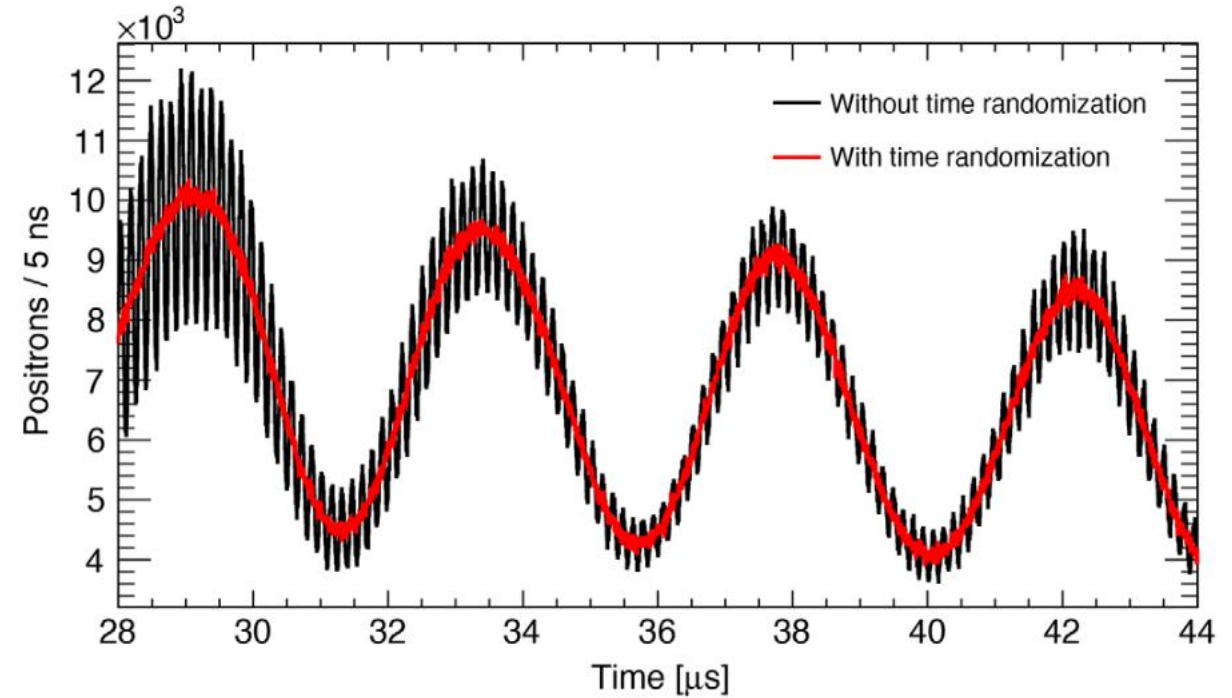
$$v_1(t) = \frac{1}{4} n(t),$$

$$v_2(t) = \frac{1}{4} n(t).$$



Finite beam length

- Individual calorimeters see has oscillation with frequency ω_c caused by bunch distribution
- Add time offset uniformly distributed between $-T_c/2, T_c/2$
- With time bunch decoheres because of momentum spread of initial beam
- Used to calculate momentum distribution \rightarrow corresponds to equilibrium radius
- Used to calculate electric field correction



Electric field correction

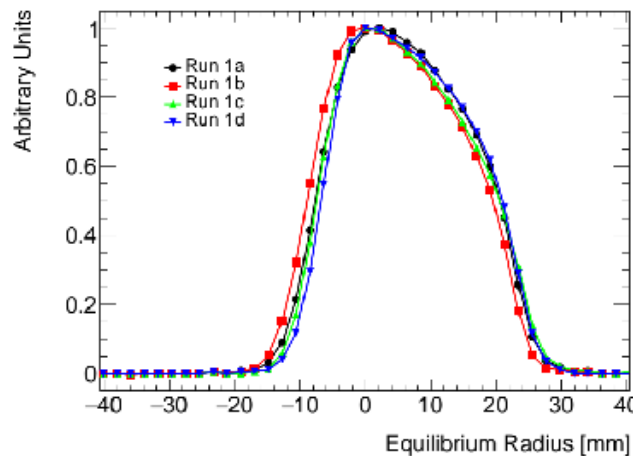
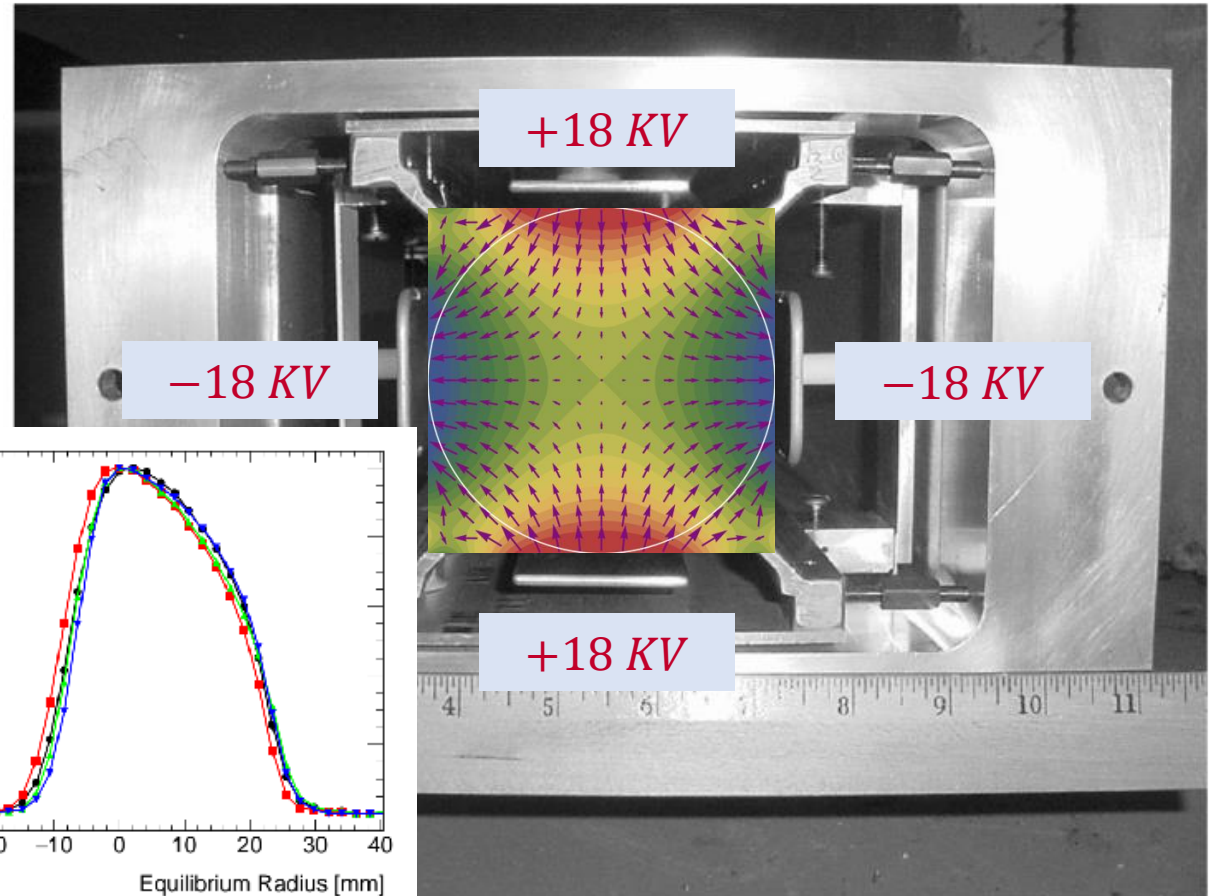
- Off-center beam sees electric field

$$\vec{\omega}_a = \frac{e}{m} \left[a_\mu \vec{B} - a_\mu \left(\frac{\gamma}{\gamma + 1} \right) (\vec{\beta} \cdot \vec{B}) \vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

- Correction given by

$$C_e = -2n(1 - n)\beta^2 \frac{\langle x_e^2 \rangle}{R_0^2}$$

- n given by ESQ HV settings
- β known from magic momentum
- R_0 nominal orbit radius



Pitch correction

- Muons have transversal momentum (pitch)

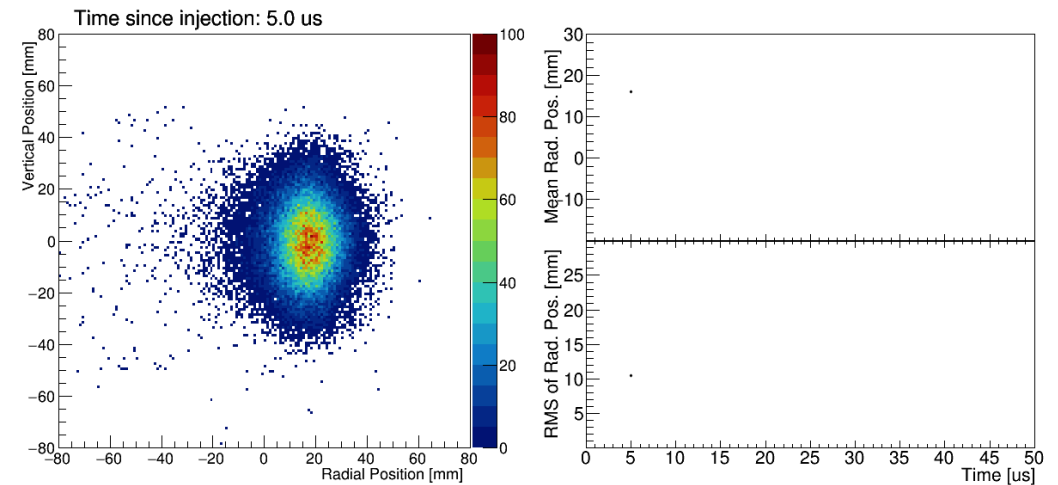
$$\frac{e}{m} \left[a_\mu \vec{B} - a_\mu \left(\frac{\gamma}{\gamma + 1} \right) (\vec{\beta} \cdot \vec{B}) \vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

- Vertical beam motion simulated by three different beam dynamics simulations

- Using tracker beam distribution as input and cross-check

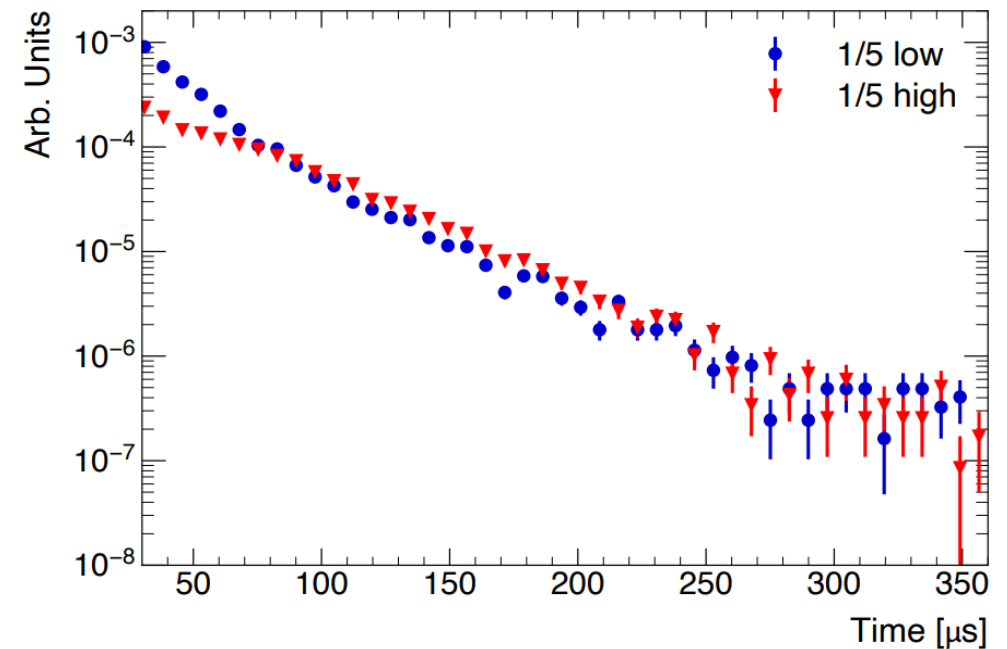
- Correction given by mean acceptance-corrected vertical amplitude

$$C_p = \frac{n}{4R_0^2} \langle A^2 \rangle$$



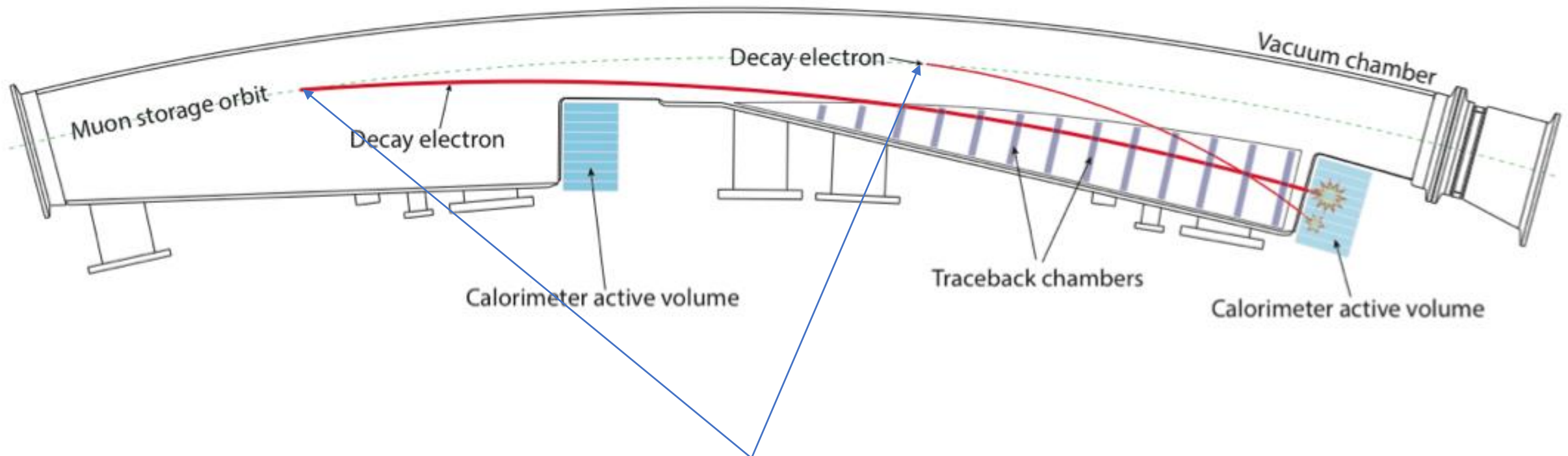
Lost muons

- Beside decay muons get lost by interaction with obstacles or collimators
- Lost muons pass through several calorimeters
- Deposited energy of a MiP with $\sim 170\text{MeV}$
- Successive calorimeter hits separated by 6.15ns
- Require measurement in three successive calorimeters to reduce random coincidences
- Monitors rate up to overall factor
- Low momentum muon lost faster
→ Early to late effect
- Needs to be corrected



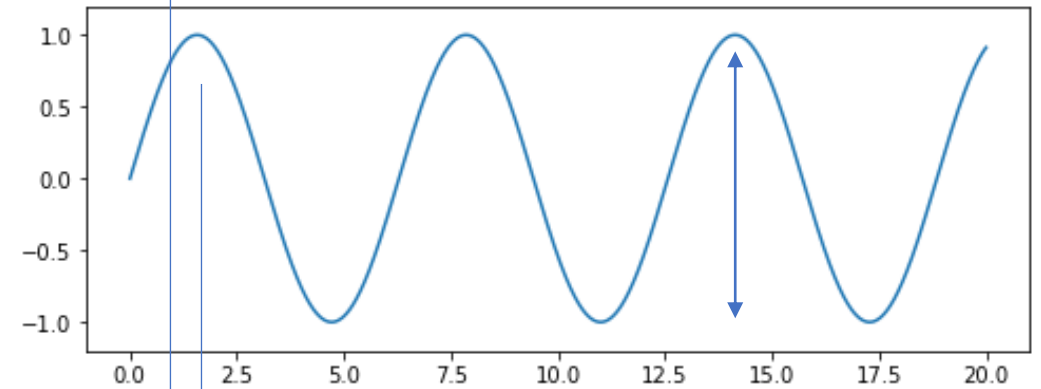
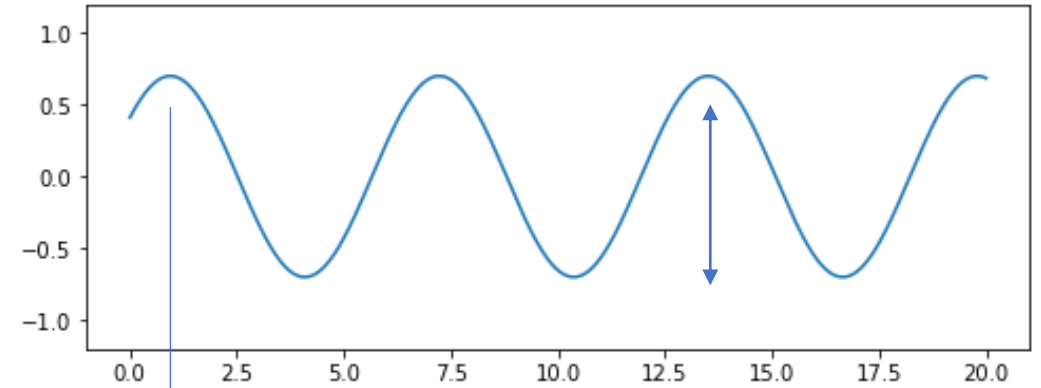
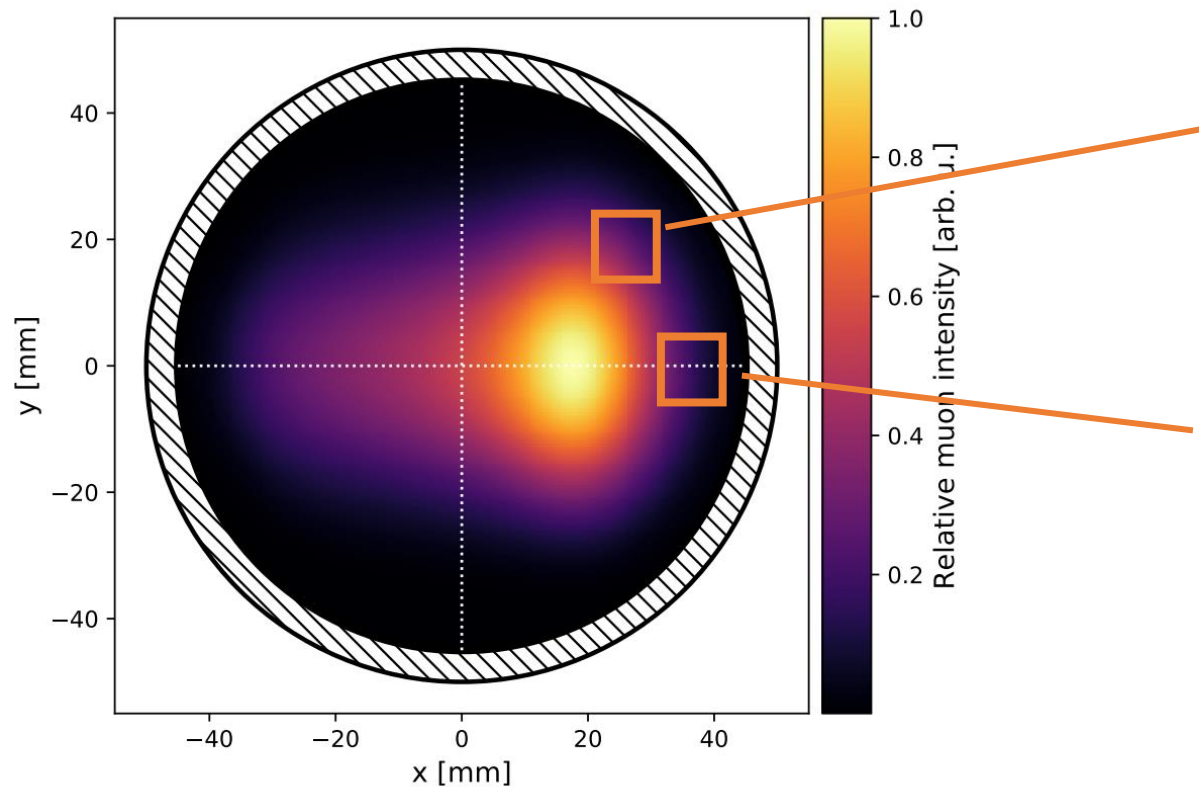
Finite Calorimeter Acceptance

- Since finite calorimeter acceptance we are sensitive to muon decay position



Muon spin has precessed a bit further

Phase acceptance



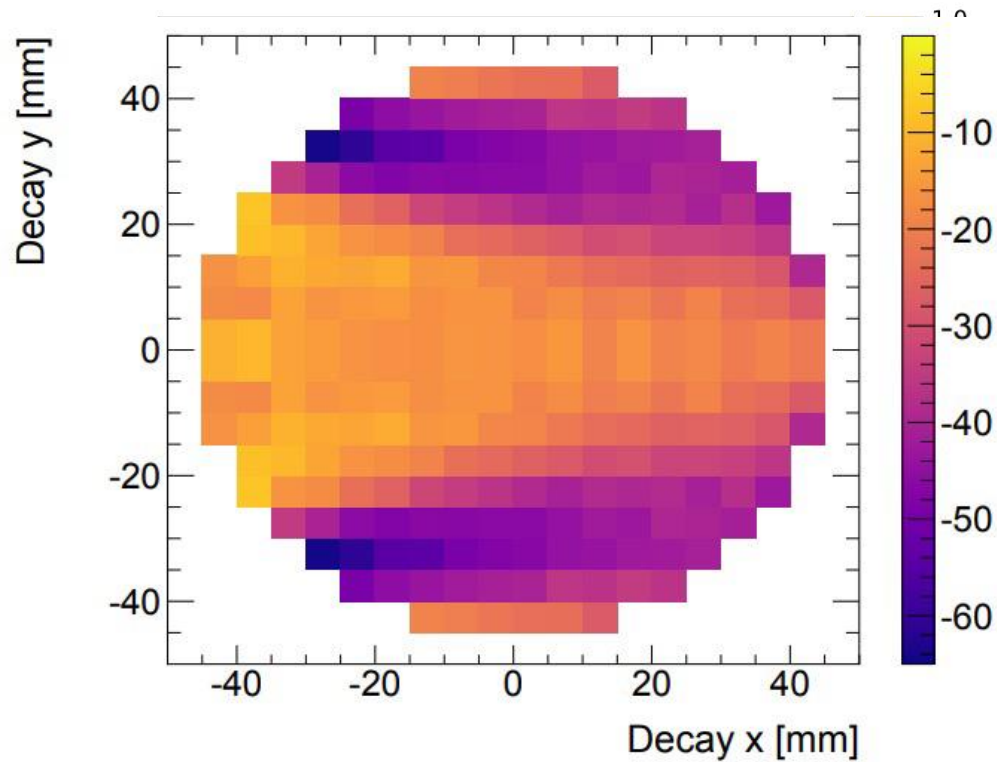
Phase shift

Asymmetry
difference

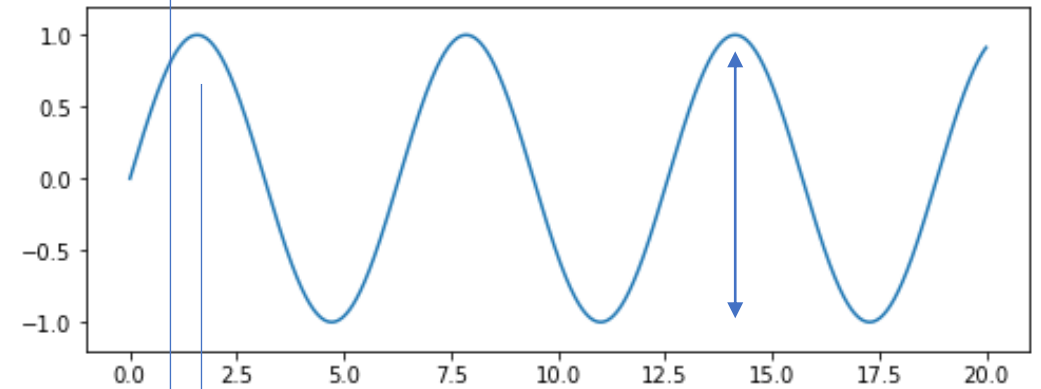
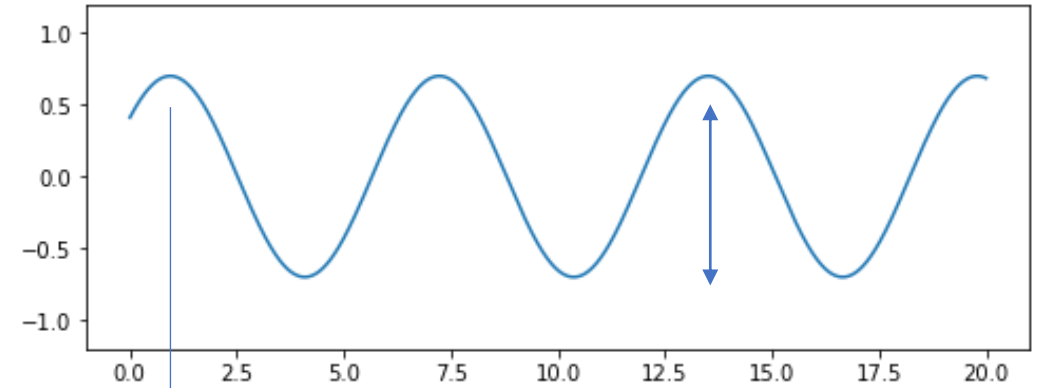
Depends on energy and time

Beam profile must be well-understood during measurement period

Phase acceptance



Detected Phase [mrad]



Phase shift

Asymmetry
difference

Depends on energy and time

Beam profile must be well-understood during measurement period

Calorimeter requirements

Fraction of positrons above a threshold energy in a calorimeter is given by

$$f(t) \propto \int_{E_{\text{thresh}}}^{E_{\text{max}}} N_0 e^{-\frac{t}{\gamma\tau}} N(E) [1 + A(E) \cos(\omega_a t - \phi(E))] dE$$

But can be written as an effective function

$$f(t) \propto N_0 e^{-\frac{t}{\gamma\tau}} [\langle N \rangle_{\text{thresh}} + \langle A \rangle_{\text{thresh}} \cos(\omega_a t - \langle \phi \rangle_{\text{thresh}})]$$

Any remaining time dependence of $\langle \phi \rangle_{\text{thresh}}$ will bias ω_a !

$$\cos(\omega_a t - \langle \phi \rangle_{\text{thresh}}(t)) \approx \cos \left[\left(\omega_a - \frac{d \langle \phi \rangle_{\text{thresh}}}{dt} \right) t - \langle \phi \rangle_{\text{thresh}}(0) \right]$$

Early to late effect

Calorimeter requirements

Assume two energy bins

$$N(t) = N_{1,0}(E_1) e^{-\frac{t}{\gamma_1 \tau_1}} [1 + A_1(E_1) \cos(\omega_a t - \phi(E_1))] \\ + N_{2,0}(E_2) e^{-\frac{t}{\gamma_2 \tau_2}} [1 + A_2(E_2) \cos(\omega_a t - \phi(E_2))]$$

Phase of summed signal

$$\tan(\phi_{\text{sum}}) = \frac{N_{1,0}(E_1) e^{-\frac{t}{\gamma_1 \tau_1}} A_1(E_1) \sin(\phi(E_1)) + N_{2,0}(E_2) e^{-\frac{t}{\gamma_2 \tau_2}} A_2(E_2) \sin(\phi(E_2))}{N_{1,0}(E_1) e^{-\frac{t}{\gamma_1 \tau_1}} A_1(E_1) \cos(\phi(E_1)) + N_{2,0}(E_2) e^{-\frac{t}{\gamma_2 \tau_2}} A_2(E_2) \cos(\phi(E_2))}$$

Any differential change between both energy groups will bias the frequency if it is time dependent!

- different storage times for different muon energies, phase-space dependent loss rates
- Detector gain change: $A_{1,2}$ are energy-dependent
- Detector pile up: wrong energy reconstruction

Pitch correction

- Muons have transversal momentum components

$$\Delta\vec{\omega}_{a,\text{pitch}} = -a_\mu \frac{e}{m} \left(\frac{\gamma}{\gamma + 1} \right) (\vec{\beta} \cdot \vec{B}) \vec{\beta}$$

- Transversal component oscillates with $\omega_y = \sqrt{n}\omega_c$
- Effect mainly averages out to first order, but second order effect is

$$\left\langle \frac{\Delta\omega_a}{\omega_a} \right\rangle_{\text{pitch}} = -\frac{\langle \psi^2 \rangle}{2} = -\frac{n \langle y^2 \rangle}{2R_0^2}$$

- Introduces always a negative bias
- Correction can be derived from measurements of the muon beam distribution

Muon beam dynamics in storage ring

- Electrostatic quadrupoles imprint harmonic potential around their central position
- Muon storage close to central position
 - Perturbative approach
- Newton's second law and Lorentz force

$$\frac{d\vec{p}}{dt} = e \left(\vec{E} + \frac{d\vec{v}}{dt} \times \vec{B} \right)$$

- Three differential equations
- Harmonic oscillator in vertical direction
- Harmonic oscillator in horizontal direction

$$\omega_y = \sqrt{n} \omega_c$$

$$\omega_x = \sqrt{1 - n} \omega_c$$

Muon beam dynamics in storage ring

- Electrostatic quadrupoles imprint harmonic potential around their central position
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- Three differential equations
- Harmonic oscillator in vertical direction
- Harmonic oscillator in horizontal direction
- n depends on quadrupole HV settings

$$\omega_y = \sqrt{n} \omega_c$$

$$\omega_x = \sqrt{1 - n} \omega_c$$

Muon beam dynamics in storage ring

- Electrostatic quadrupoles imprint harmonic potential around their central position
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 - Perturbative approach
- Newton's second law and Lorentz force

$$\frac{d\vec{p}}{dt} = e \left(\vec{E} + \frac{d\vec{v}}{dt} \times \vec{B} \right)$$

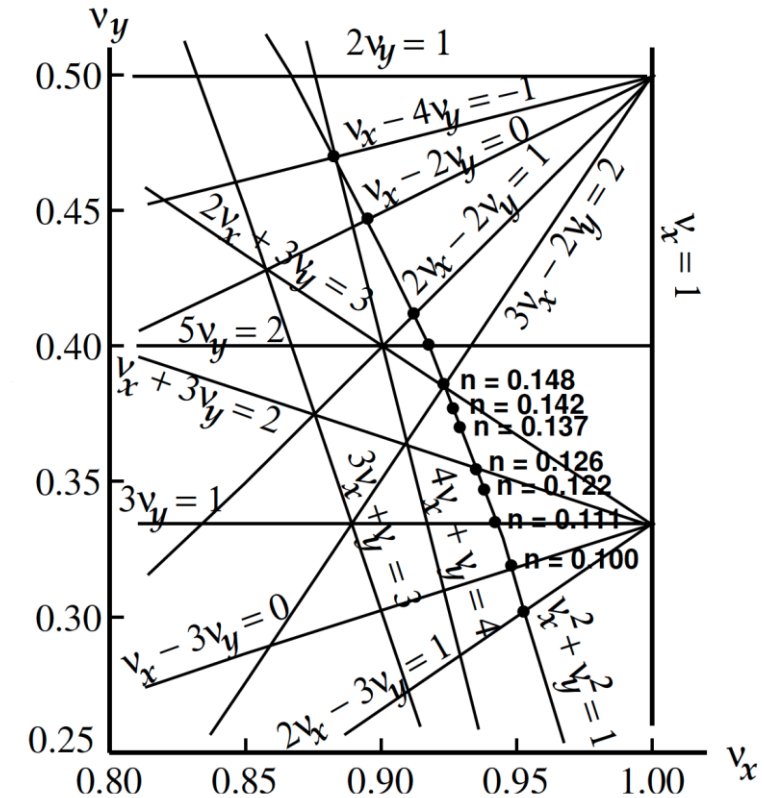
- Three differential equations
- Harmonic oscillator in vertical direction
- Harmonic oscillator in horizontal direction
- n depends on quadrupole HV settings
- Resonant condition for

$$M\nu_x + N\nu_y = P \quad \text{with } M, N \in \mathbb{Z} \text{ and } P \in \mathbb{N}$$

- Avoid ω_a interference

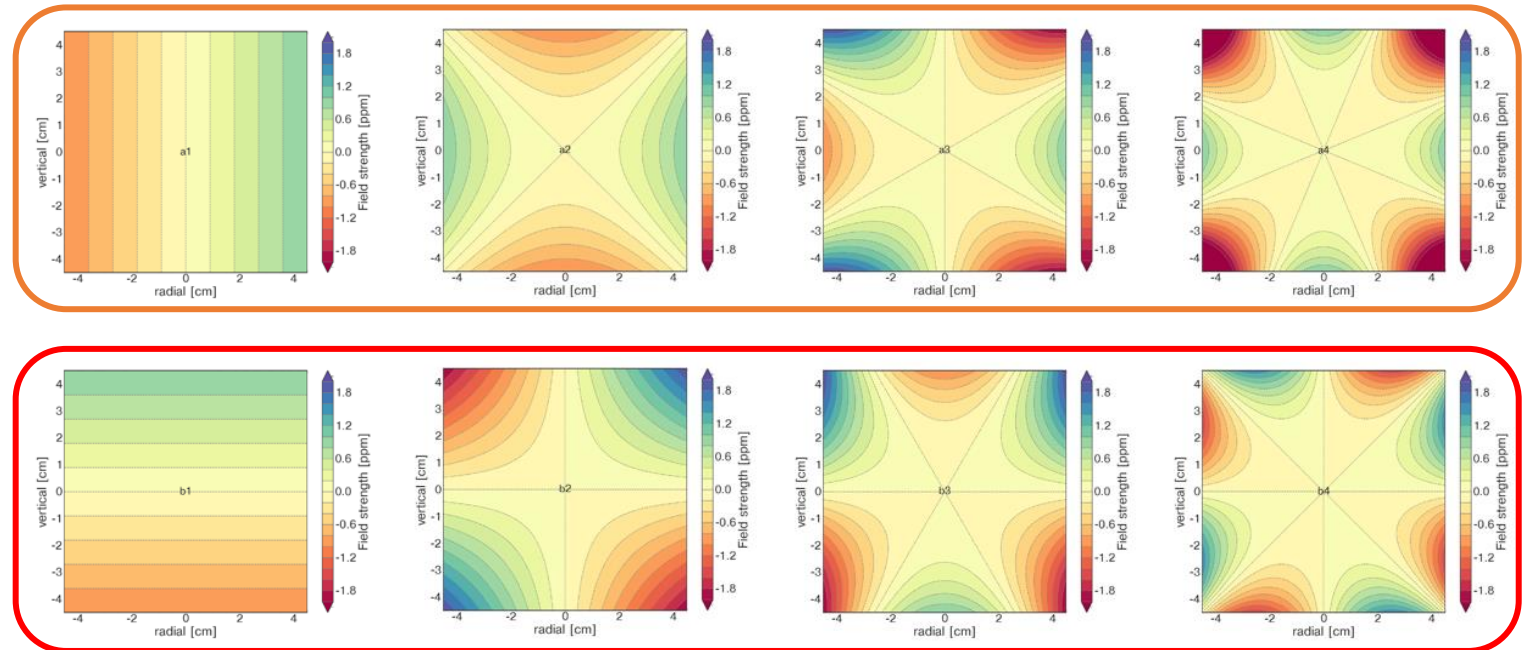
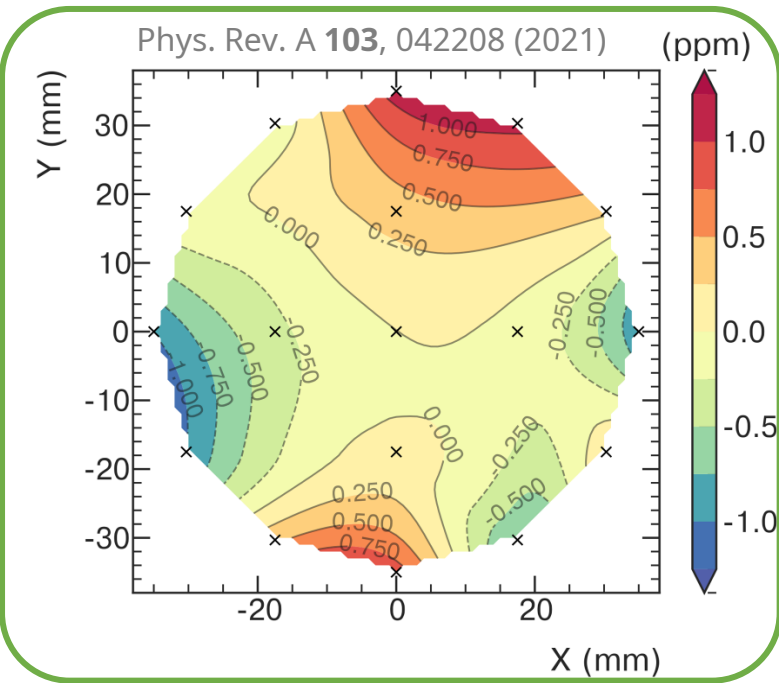
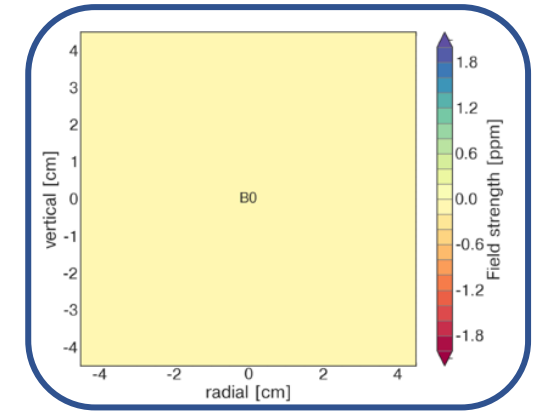
$$\omega_y = \sqrt{n} \omega_c$$

$$\omega_x = \sqrt{1-n} \omega_c$$



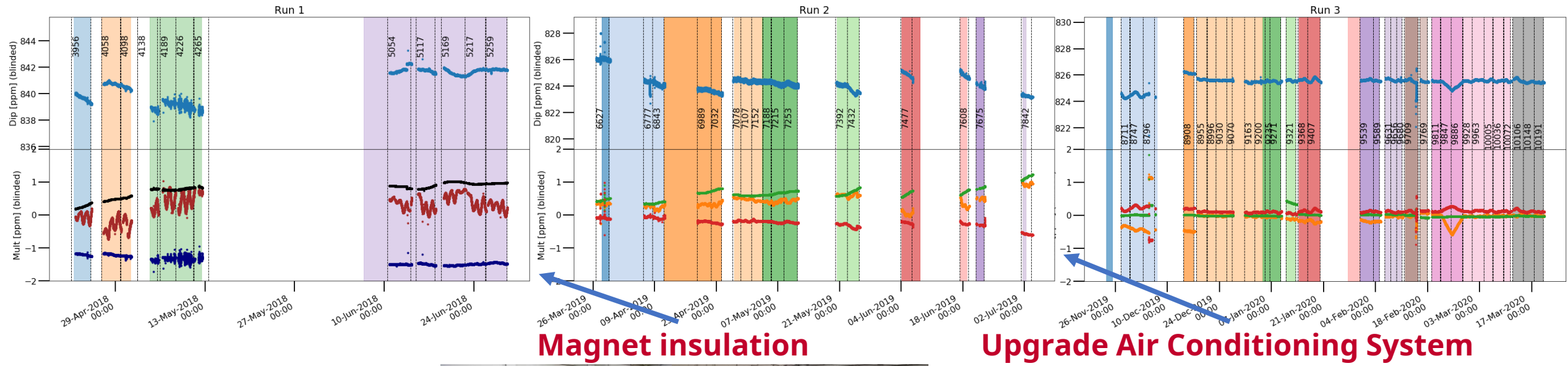
Spatial dependence of B-field

$$B(r, \theta) = B_0 + \sum_{n=1}^4 \left(\frac{r}{r_0} \right)^n [a_n \cos(n\theta) + b_n \sin(n\theta)]$$



Improved operating conditions

Magnetic field stability



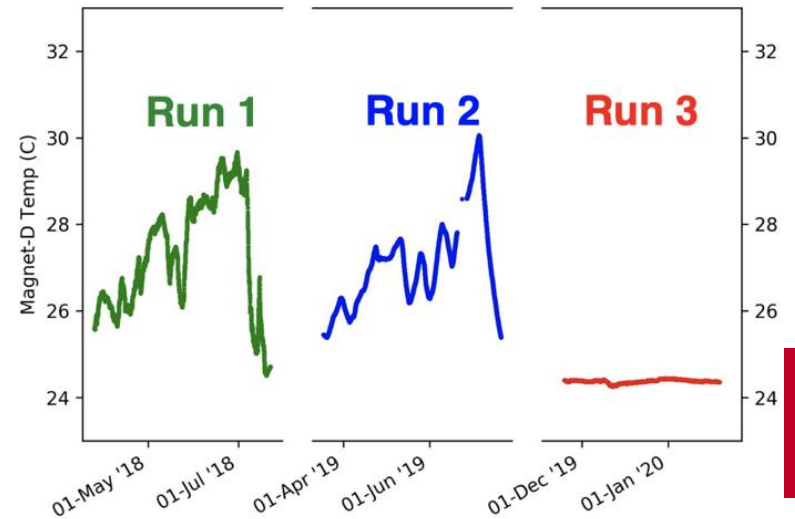
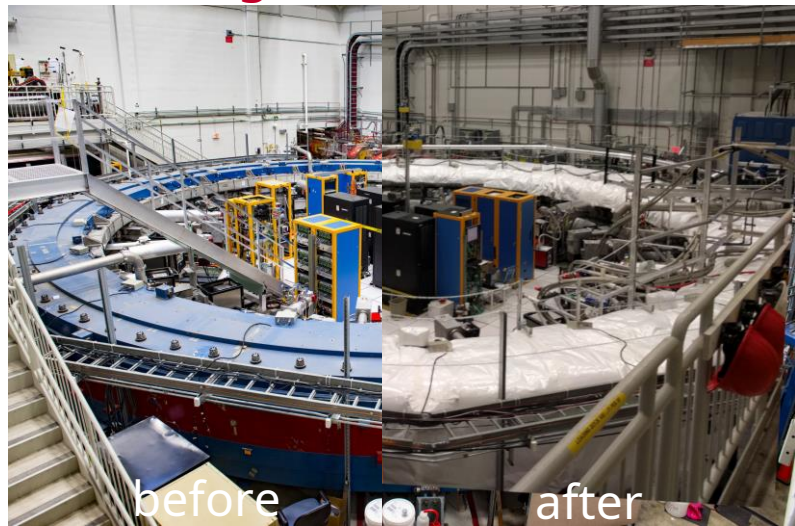
Magnet insulation

Upgrade Air Conditioning System

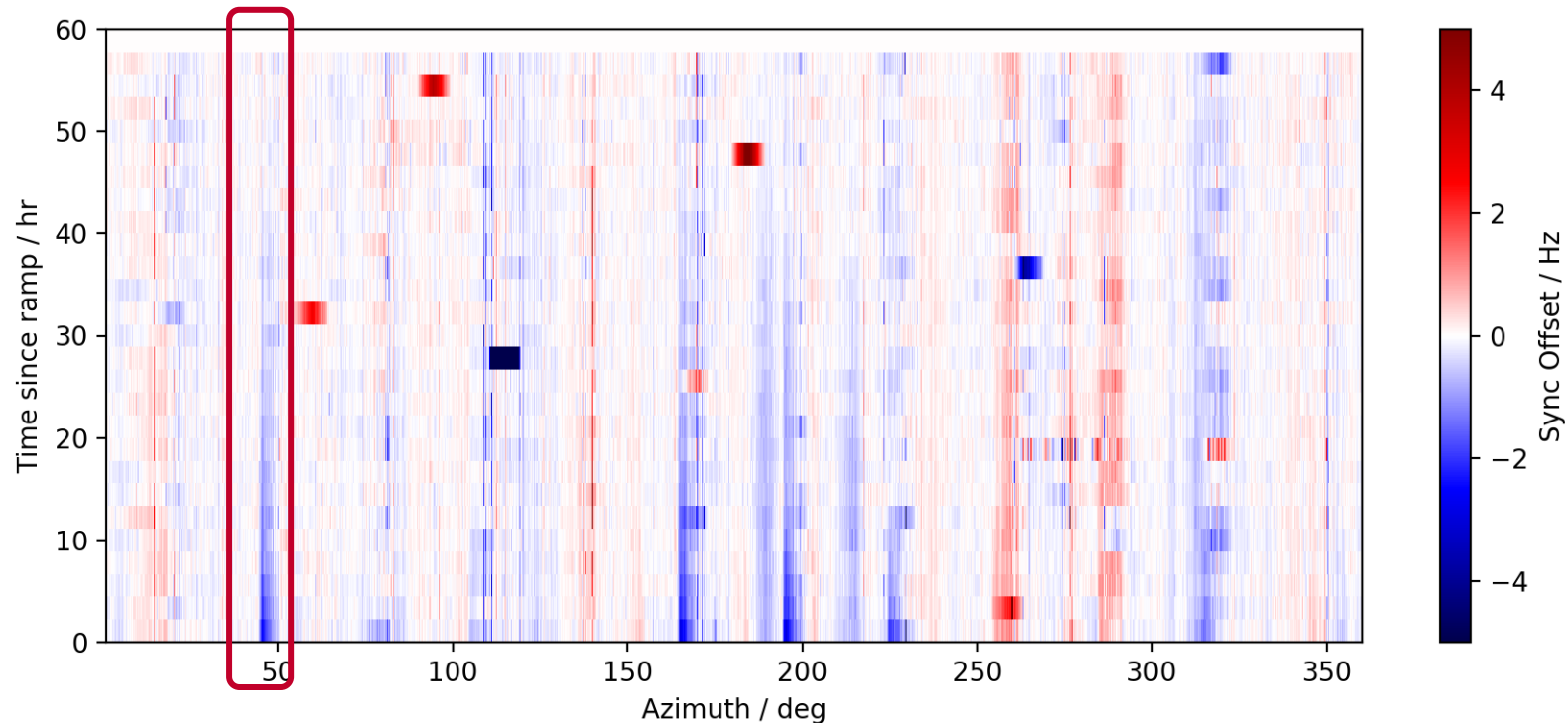
Hall temperature changes caused field changes

See diurnal field variation

Influences magnetic field systematic

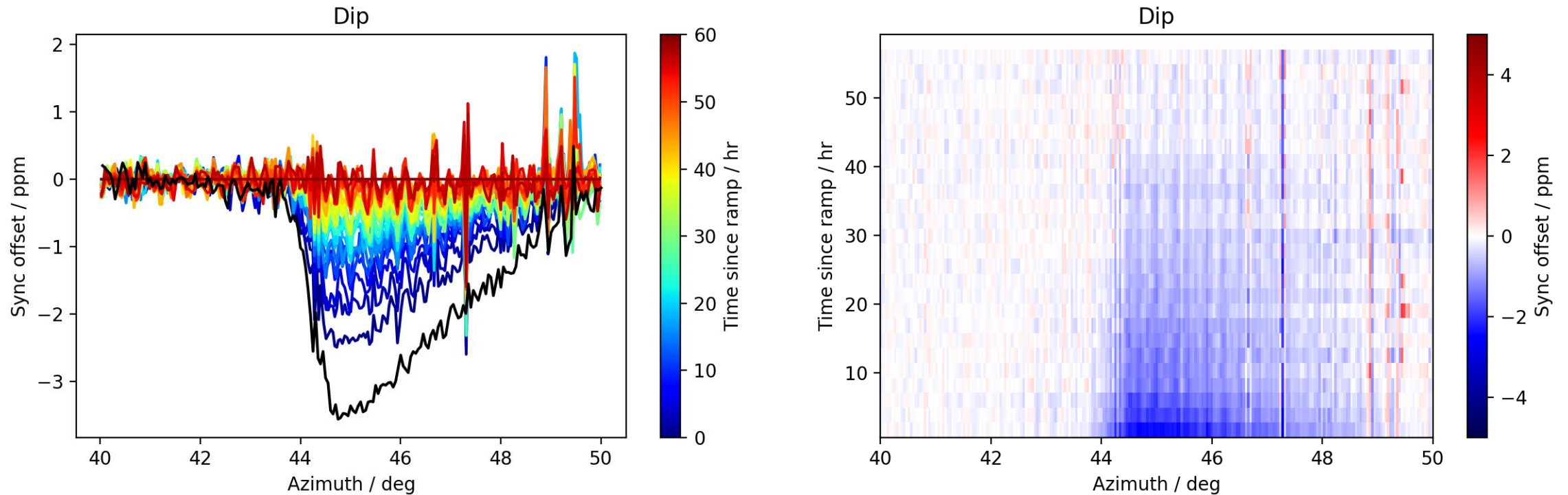


Effect of Magnet Cycling



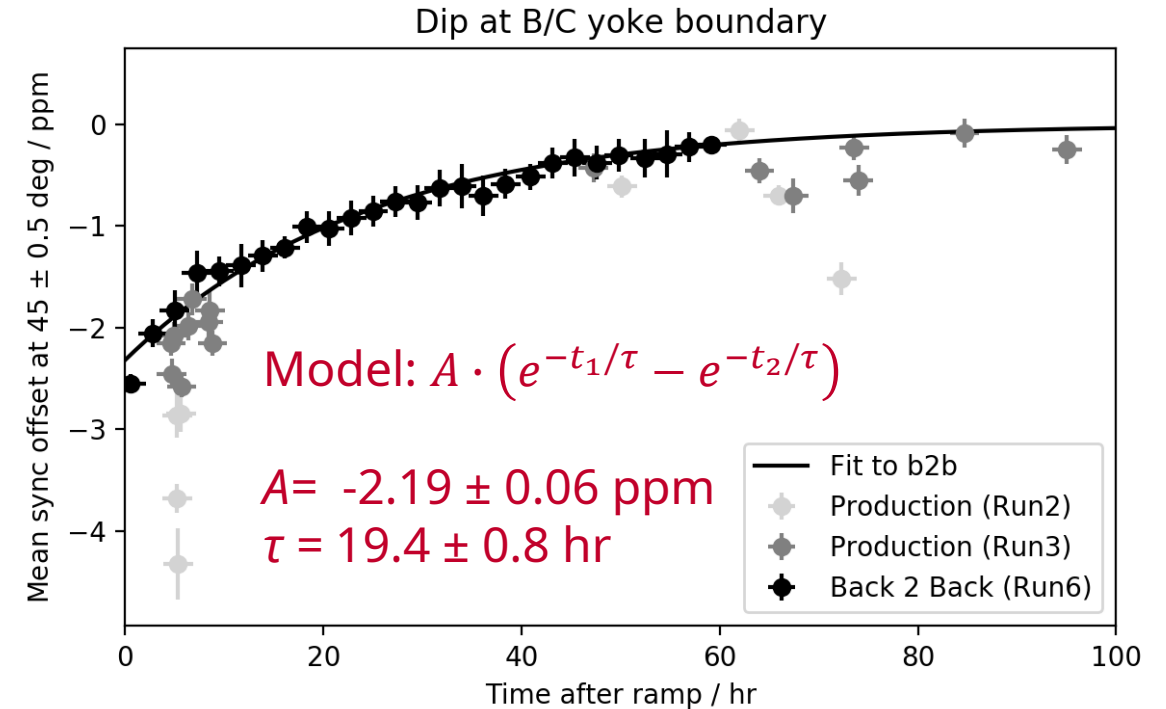
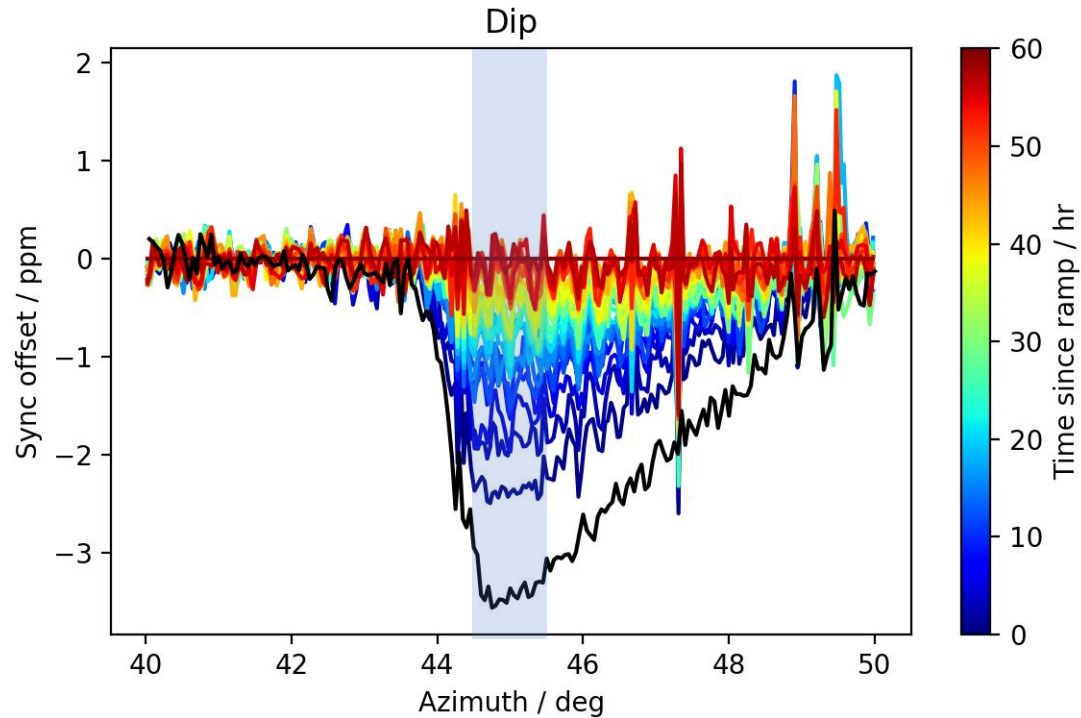
- Magnet cycling while operation necessary for cryo-maintenance, repairs, special studies, ...
- Special study: Ramp magnet on purpose, perform trolley measurements for 60 h
- Azimuthal resolved sync offset always w.r.t. the last trolley run
- Effects at 12 yoke boundaries visible
- Time dependence of magnetic field visible

Field at Yoke Boundary B/C



- Increased and time depended sync offsets at yoke boundary region
- Characteristic shape of untracked field changes
- Untracked field changes get smaller over time

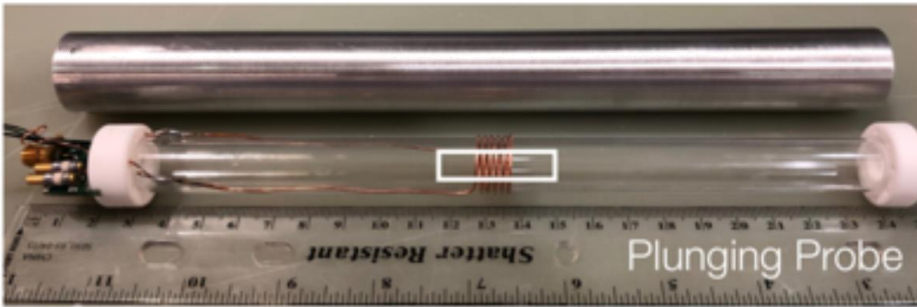
Field at Yoke Boundary B/C



- Fitting amplitude of effect with exponential function
- Time constants of ~ 20 h

Trolley Probe Calibration

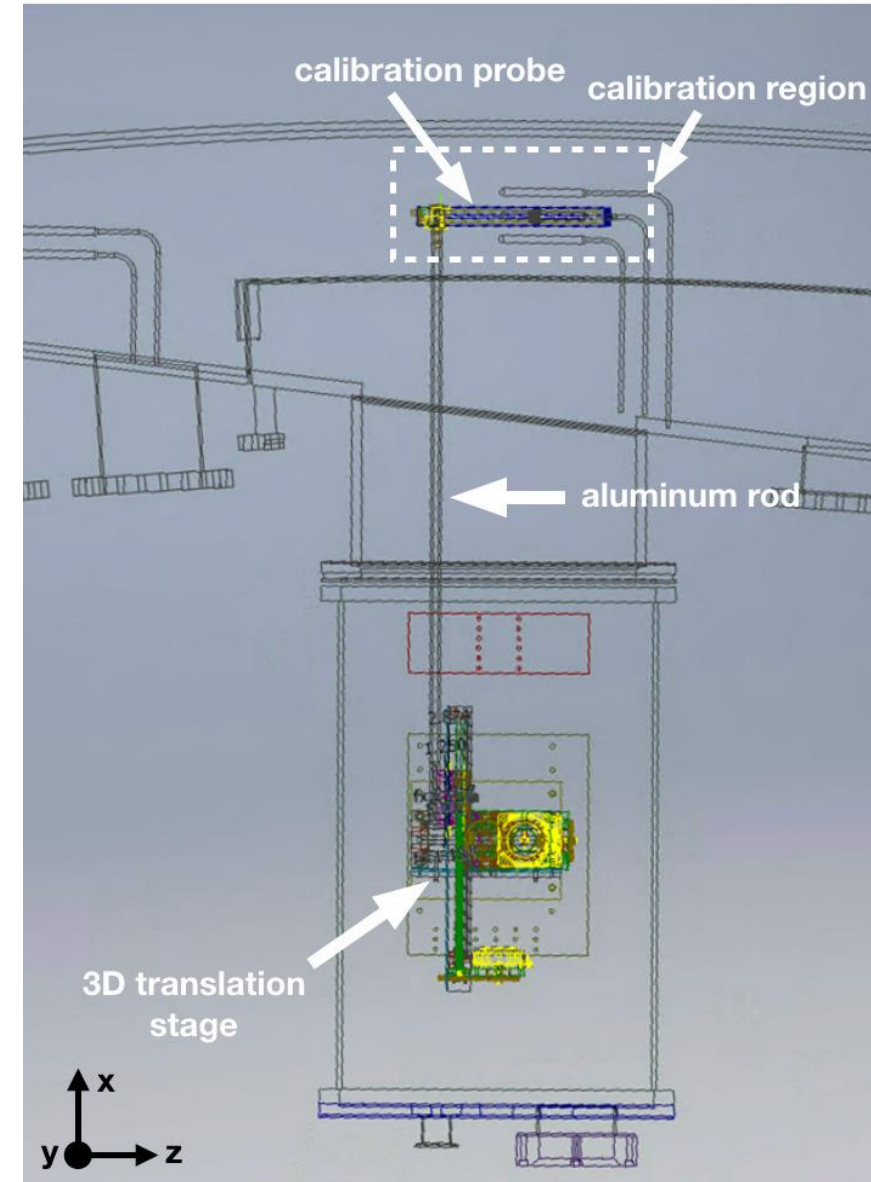
- Absolute calibrated water probe
- Cross-calibrated at Argonne National Lab test magnet



- Probe can be placed in ring by 3D translation stage
- Swap trolley and calibration probe to get calibration constant
- Derive calibration constants for each trolley probe

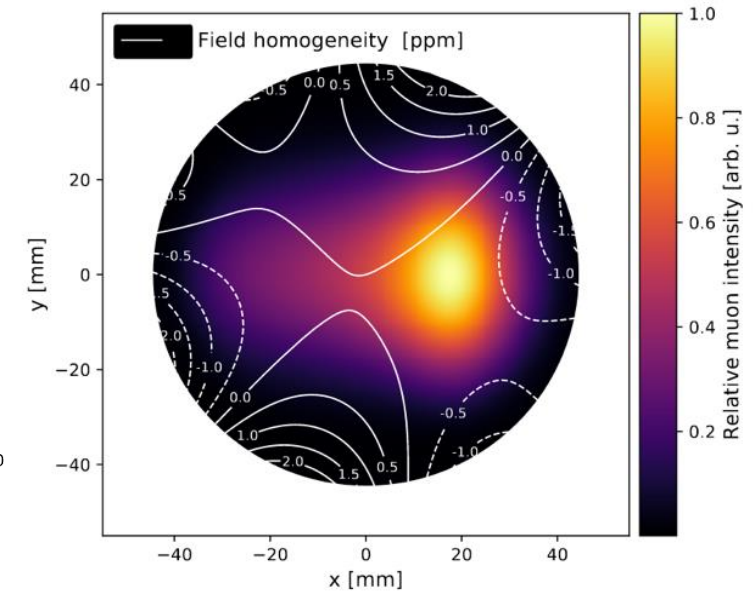
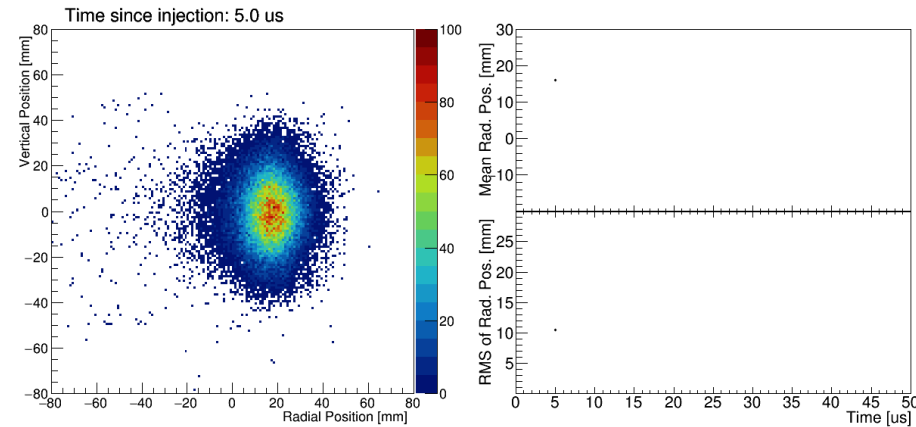
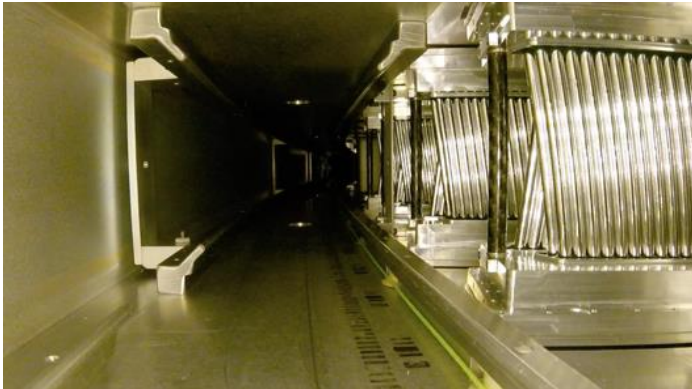
Improvements

- Study of temperature effects
- Calibration twice a year with automated procedure
- Consistent results



Muon weighted average magnetic field

Using tracker profiles and beam dynamics simulation



Improvements

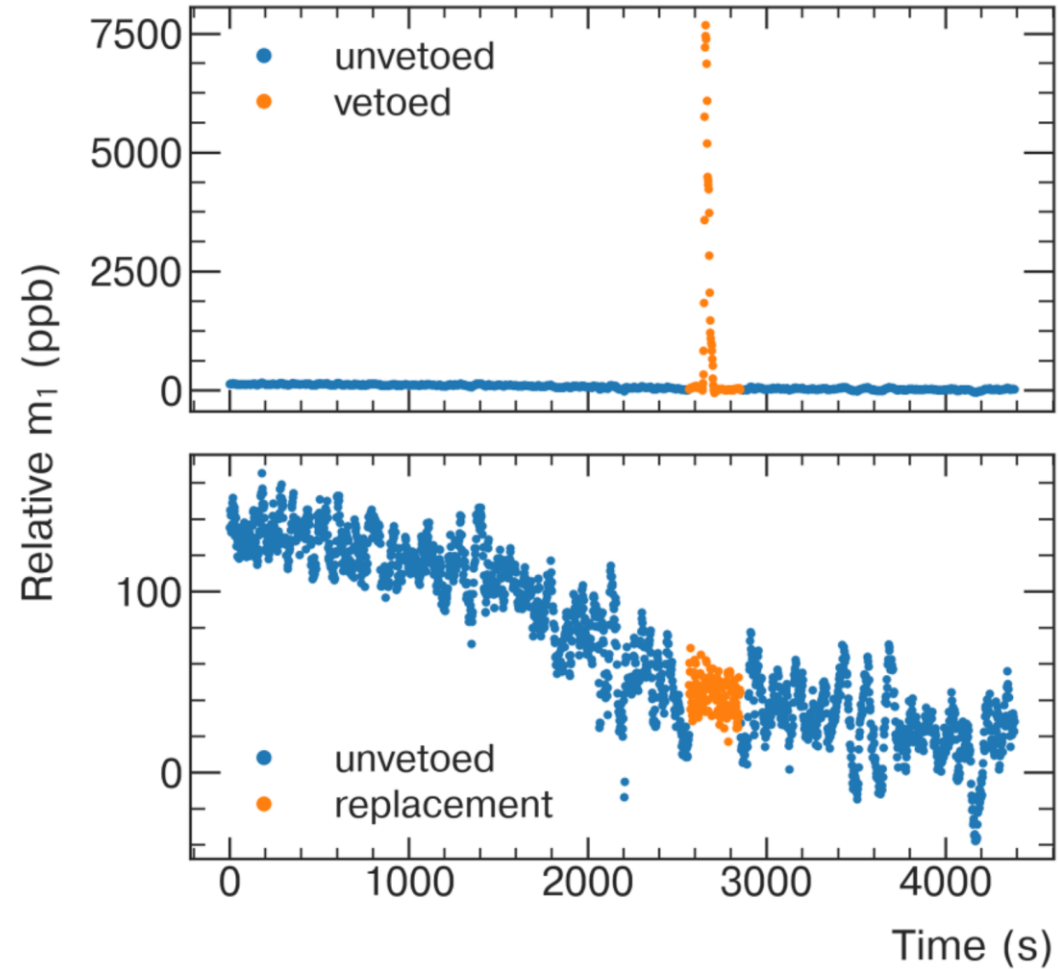
- Better placement of beam, due to replacing broken resistors
- Better placement of beam, due to stronger kick
- CBO reduction due to quad RF (run 5)

Trolley Footprint Removal

Phys. Rev. A **103**, 042208 (2021)



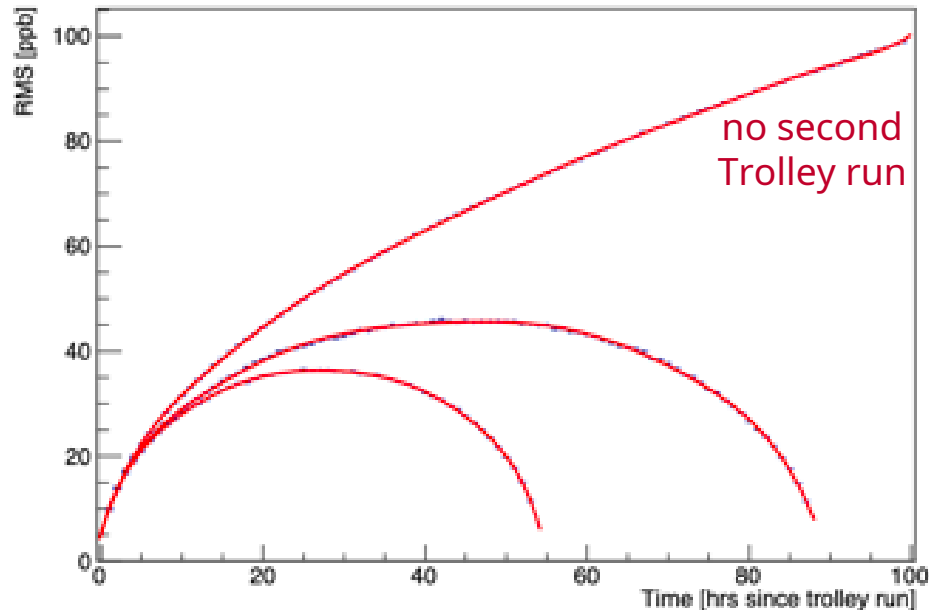
- trolley electronics disturbs field (footprint)
- veto measurements
- interpolate from neighboring probes



Tracking Uncertainty

- Fixed Probe drift: Random walk
- End point known: Brownian bridge model

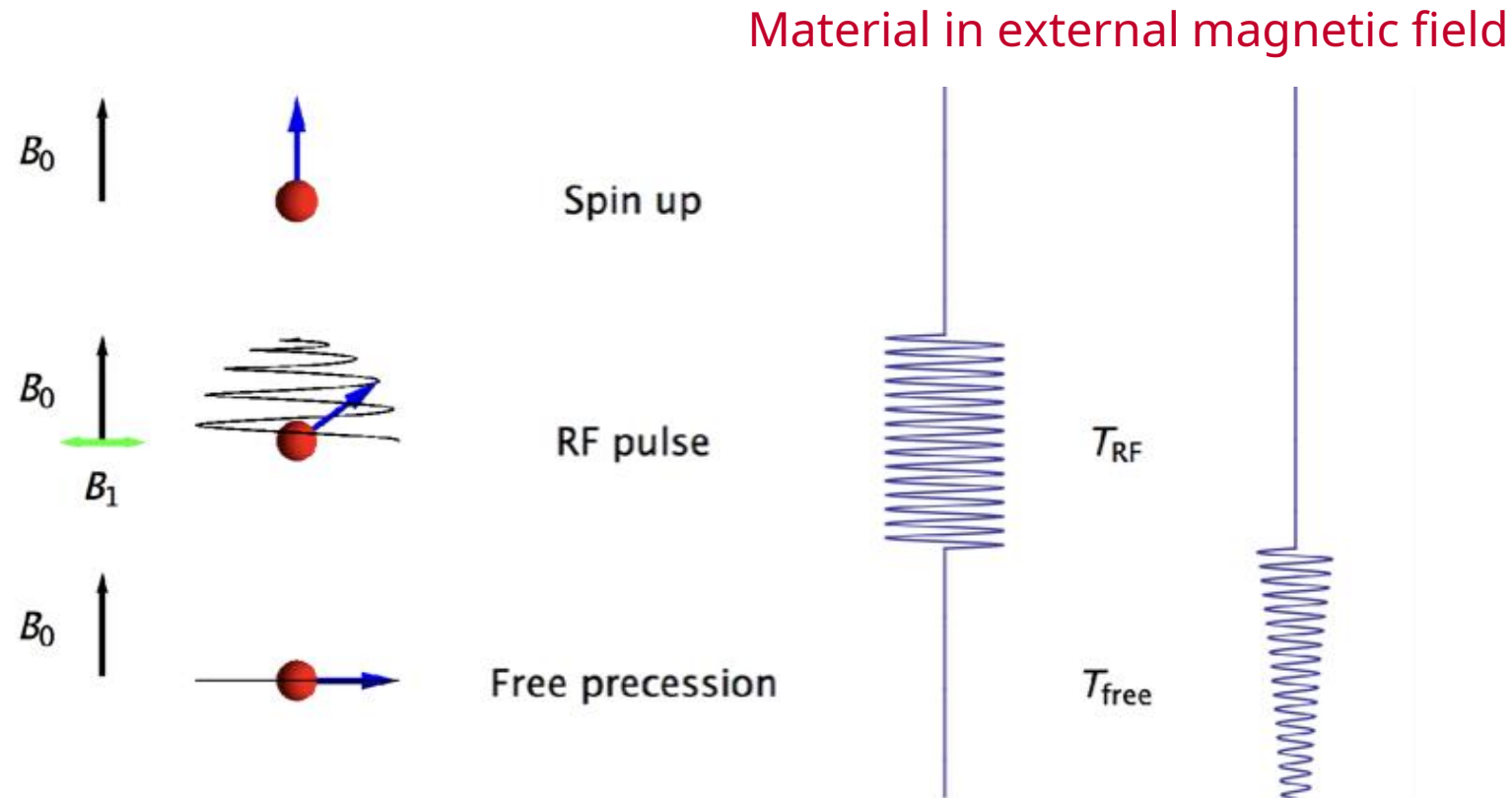
$$\sigma(t_1, t_2) = M \frac{(T - t_2)t_1}{T}$$



Improvements

- improved position determination
- continues azimuthal treatment (virtual trolley, Fourier method)
- improved trolley footprint removal
- more trolley runs
- improved field stability by temperature regulation

Nuclear Magnetic Resonance (NMR) technique



thermal equilibrium
polarization: $\sim 10^{-6}$

RF pulse perpendicular
to main field close to
proton Larmor frequency
tilts the p spin

Pick up induction signal
of precessing magnetization
with the excitation coil

NMR technique

- Lamor precession frequency

$$\omega_L = -\gamma B$$

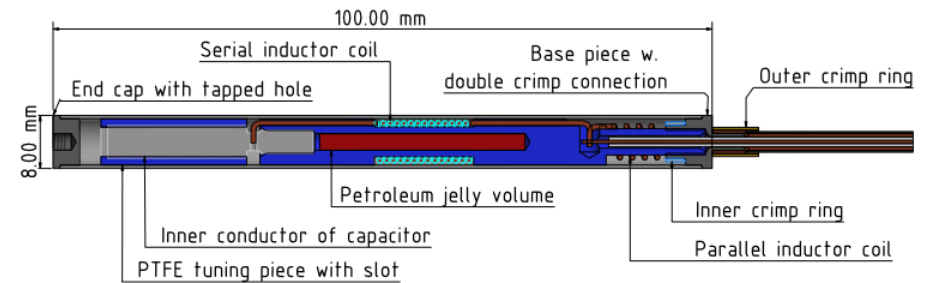
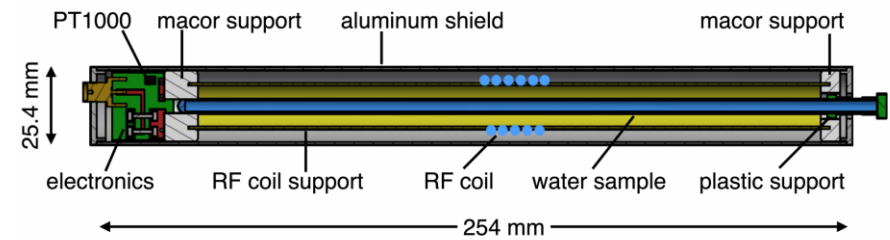
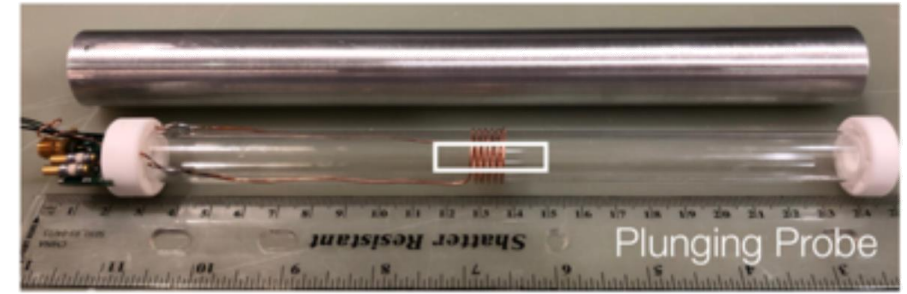
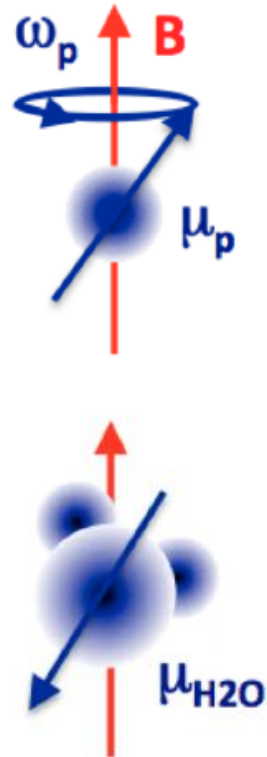
with gyromagnetic ratio γ

- Gyromagnetic ratio of free proton is $2.6752218744 \cdot 10^8 \text{ Hz/T}$

- Reference gyromagnetic ratio of pure water in spherical sample

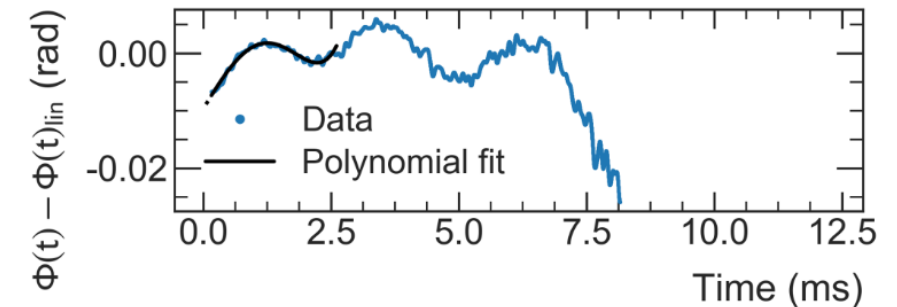
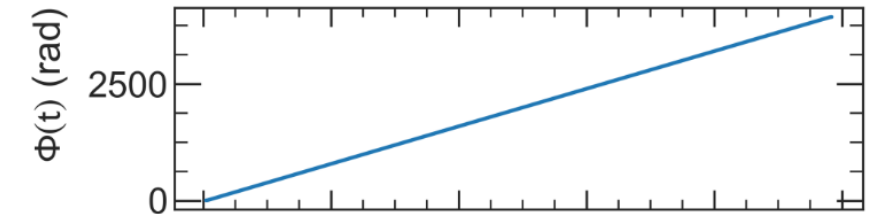
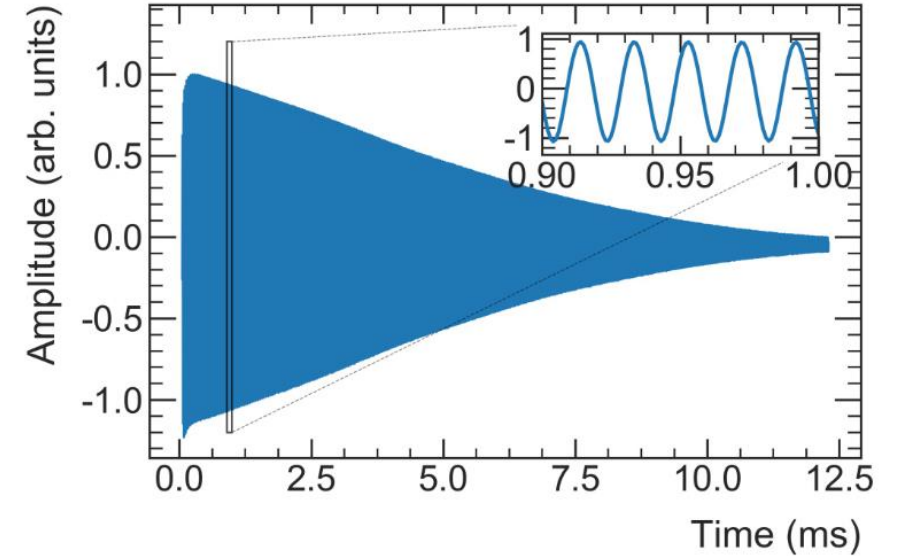
- Two types of probes

- Ultra pure water in cylinder volume for calibration
- Petroleum jelly in cylinder volume for normal measurement

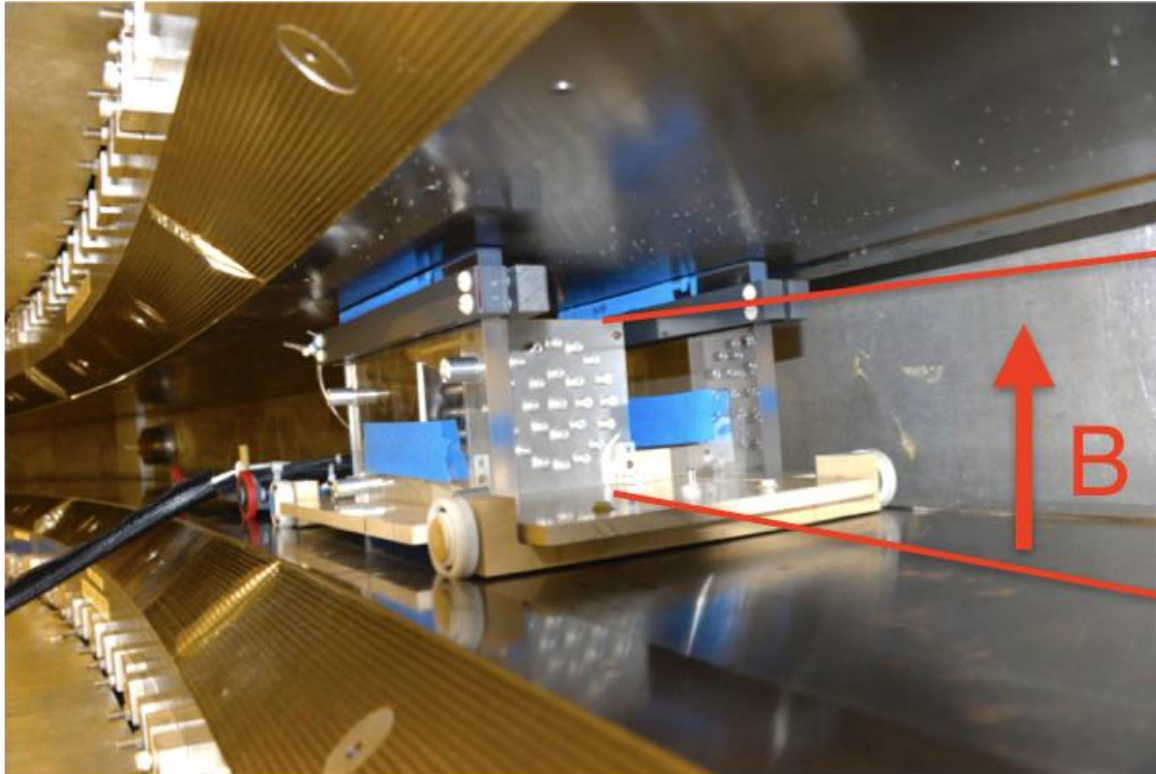


Free Induction Decay

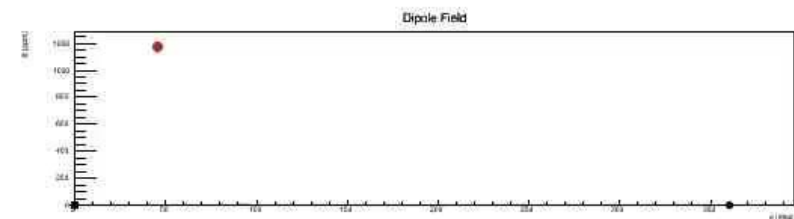
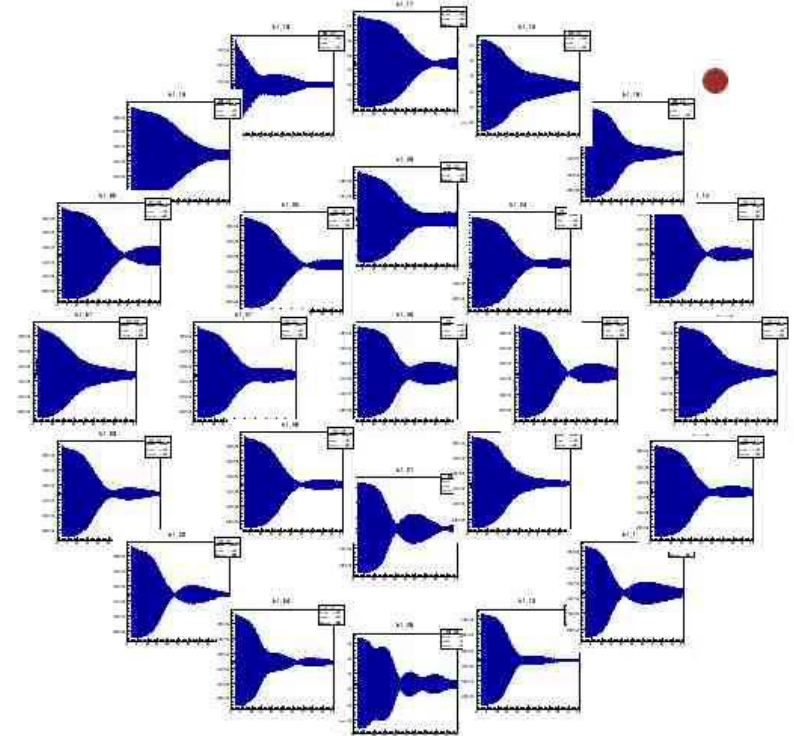
- At 1.45 T field proton spin precession frequency is about 61.79 MHz
- Mixed down frequency to ~50kHz for digitization
- Free induction decay signal oscillates at Larmor frequency
- Decoherence of spins in sample lead to envelop decay
- Using Hilbert transformation to extract phase
- Frequency is given by slope of phase at time $t=0$
- Subtract template \rightarrow measure field differences



Shimming trolley

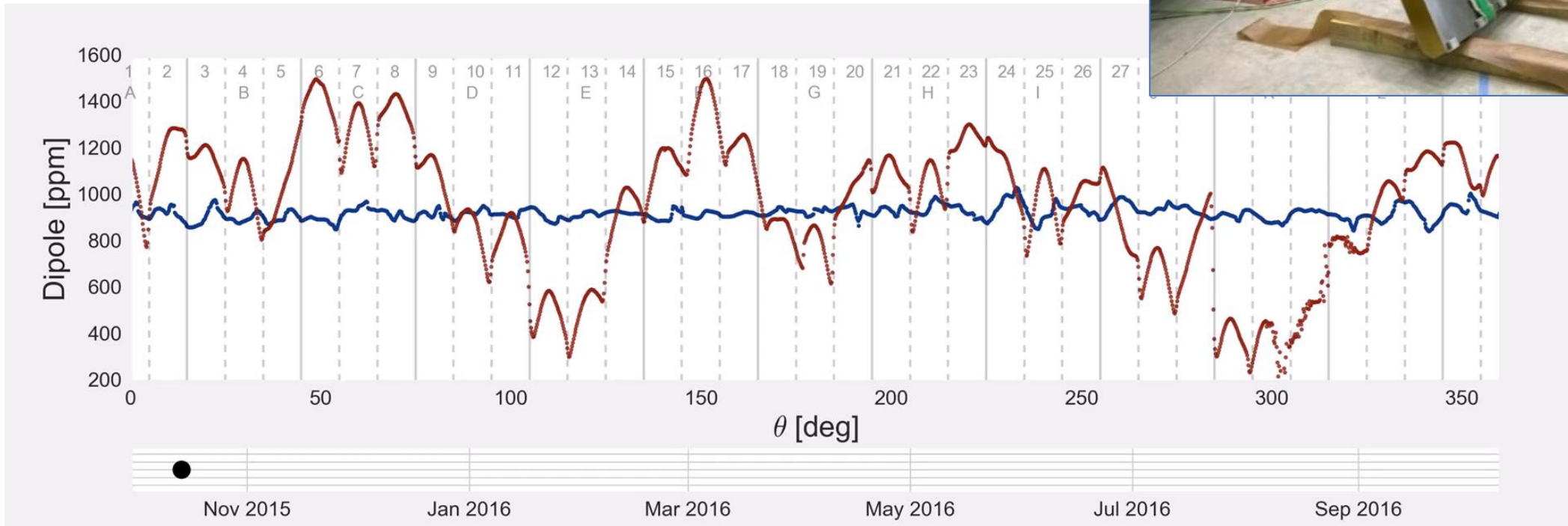


- 25 NMR probes on movable platform
- Used to measure field while assembly



Getting a homogeneous field

First adjust height and tilt of pole pieces

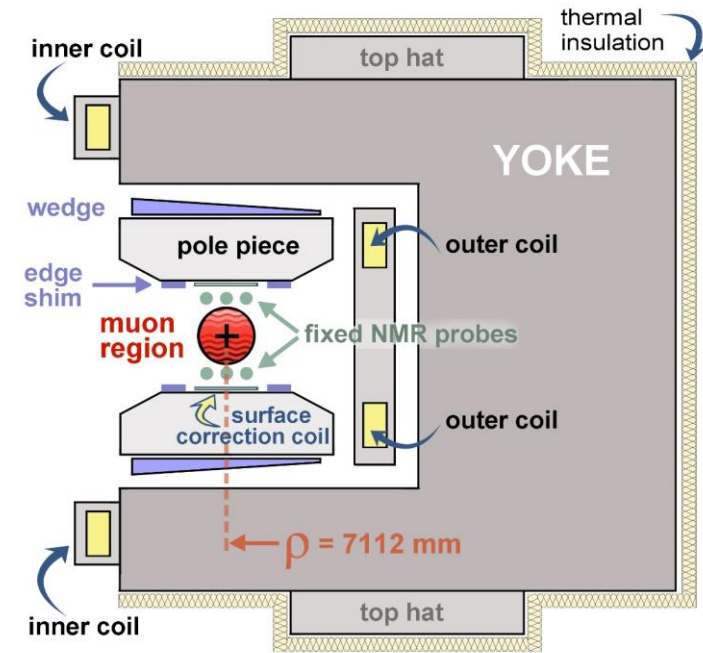


Getting a homogeneous field

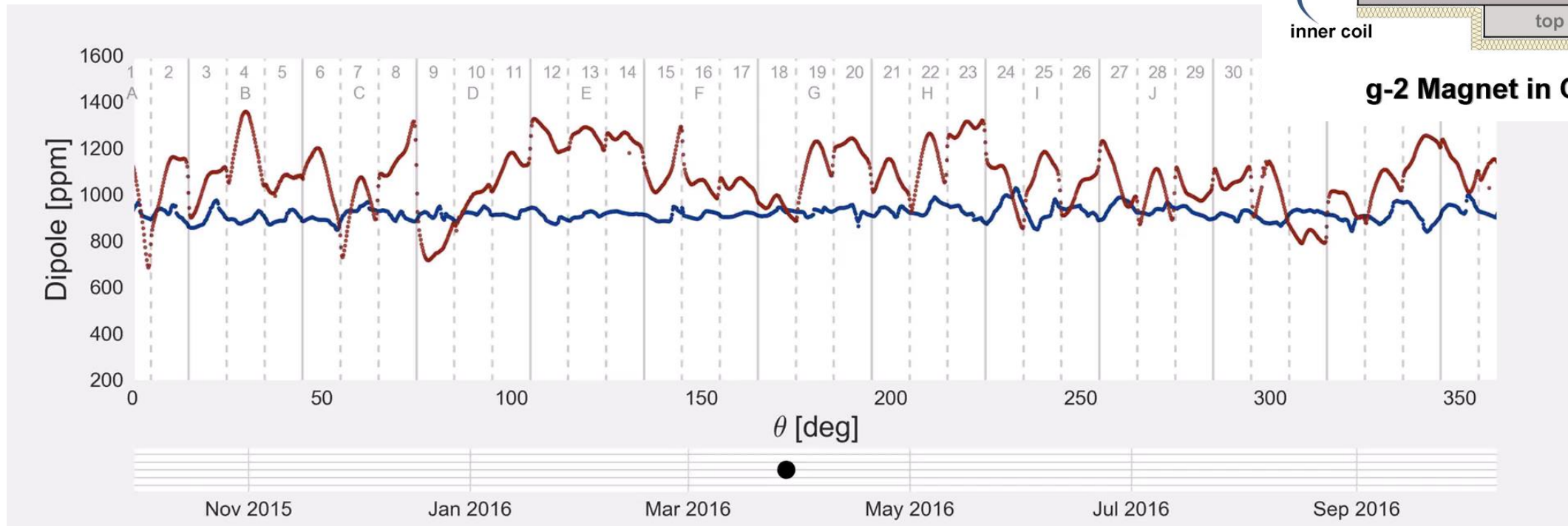
Second top hats and wedge shims

Top hats gap changes effective permeability in the magnetic circuit

Radial position of wedges to adjust dipole and compensate quadrupole

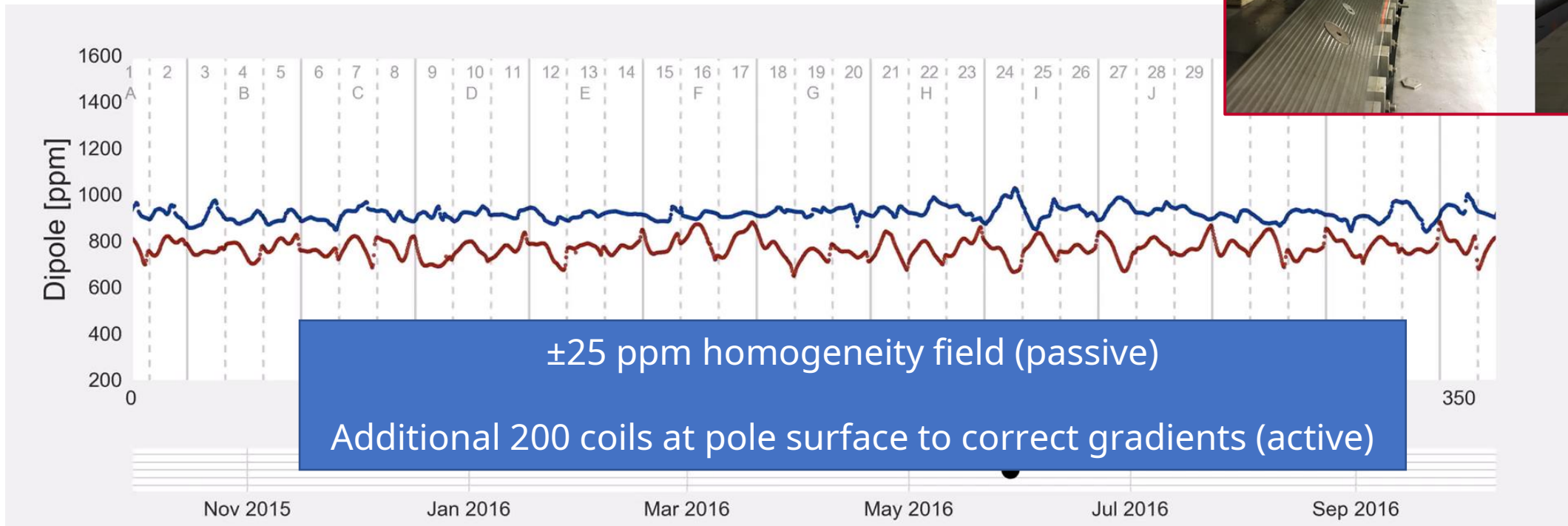
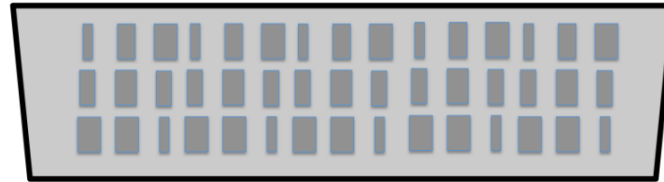


g-2 Magnet in Cross Section

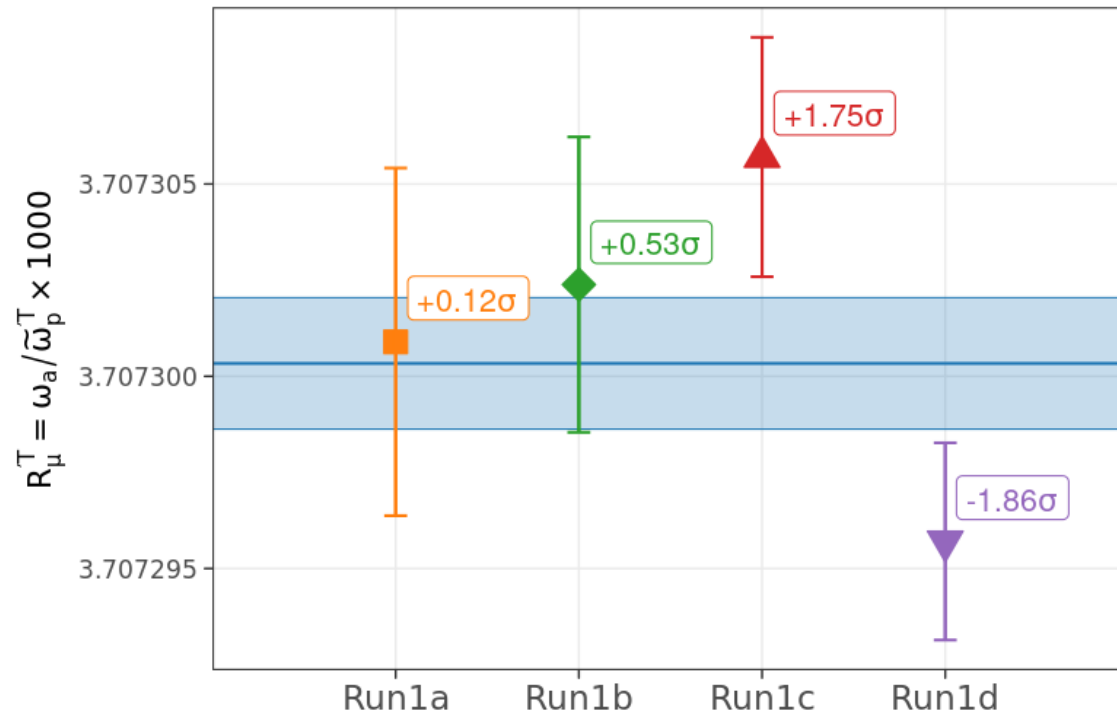


Getting a homogeneous field

Add IR laser cut iron foils



Blinded results from 4 data periods



- Correction factors and analysis depend on kicker strength and ESQ HV settings (beam tune)
- Four different settings in run 1
- Results consistent with $\chi^2/\text{ndf}=6.8/3$ $P(\chi^2)=7.8\%$
- Result still hardware blinded

Blinding of master clock

... by Greg Bock and Joe Lykken in 2018 (no members of Muon g-2 collaboration)



ω_a reference clock supposed to be at 40 MHz but slightly detuned

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}} \omega_a^{\text{meas}} (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{\text{calib}} \langle M(x, y, \phi) \omega'_p(x, y, \phi) \rangle (1 + B_k + B_q)}$$

Hardware blinding of master clock

... by Greg Bock and Joe Lykken in 2018 (no members of Muon g-2 collaboration)

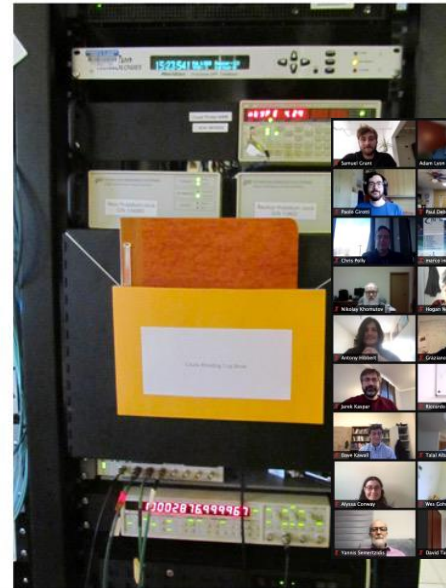


ω_a reference clock supposed to be at 40 MHz

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}} \omega_a^{\text{meas}} (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{\text{calib}} \langle M(x, y, \phi) \omega'_p(x, y, \phi) \rangle (1 + B_k + B_q)}$$

Hardware blinding of master clock

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... on February 25th, 2021!



ω_a reference clock supposed to be at 40 MHz

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}} \omega_a^{\text{meas}} (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{\text{calib}} \langle M(x, y, \phi) \omega'_p(x, y, \phi) \rangle (1 + B_k + B_q)}$$

Hardware blinding of master clock

... by Greg Bock and Joe Lykken in 2018 (no members of Muon g-2 collaboration)



... on February 25th, 2021!

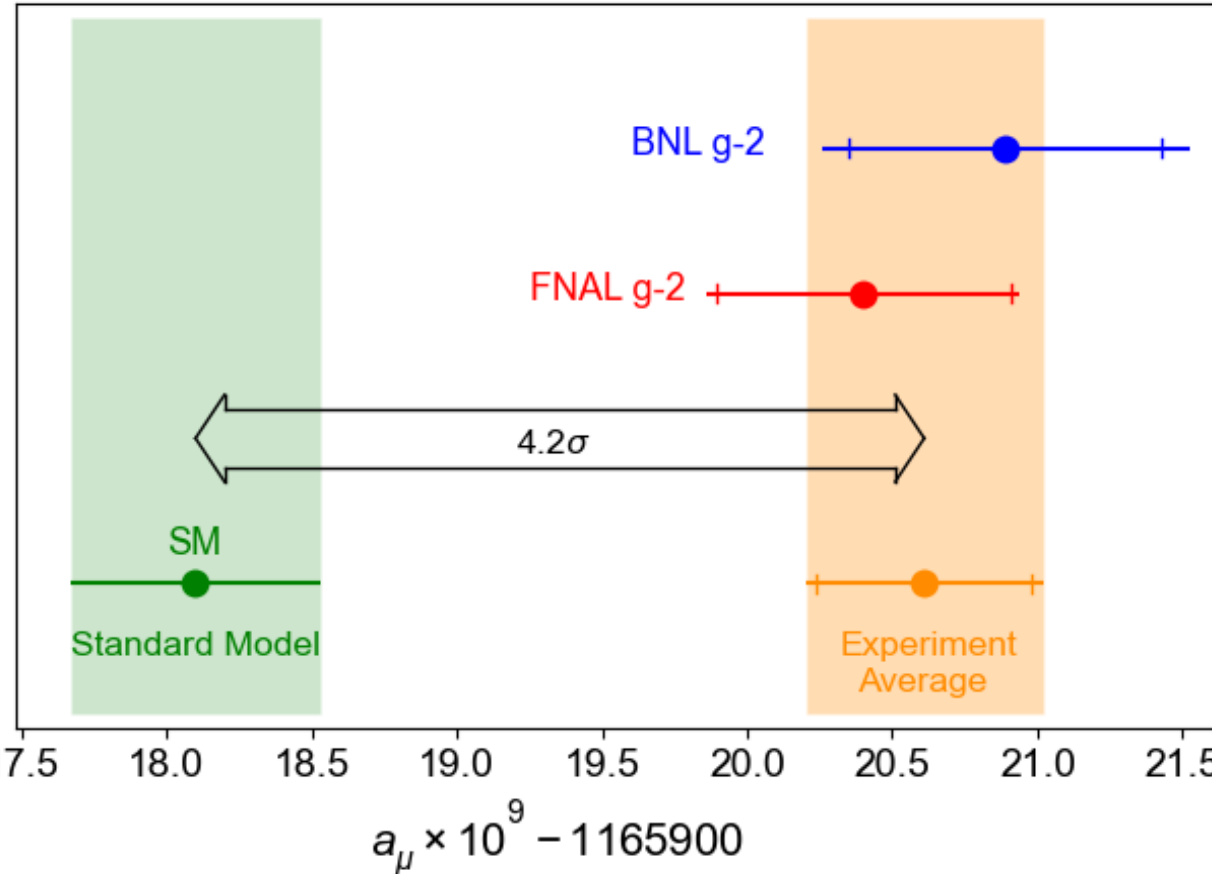


ω_a reference clock supposed to be at 40 MHz but was set to 39.997 844 MHz

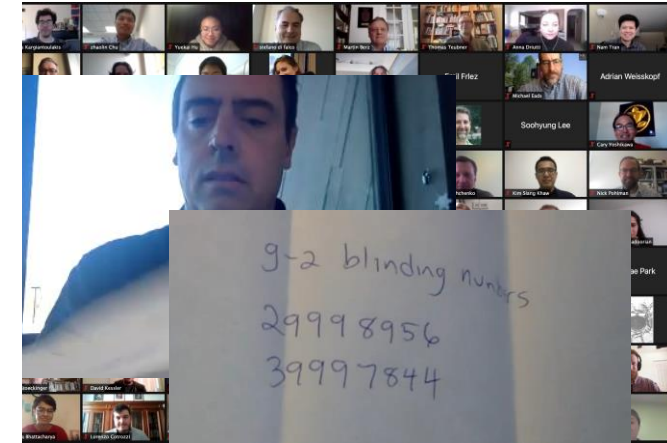
$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{f_{\text{clock}} \omega_a^{\text{meas}} (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{\text{calib}} \langle M(x, y, \phi) \omega'_p(x, y, \phi) \rangle (1 + B_k + B_q)}$$

Hardware blinding of master clock

... by Greg Bock and



ation)

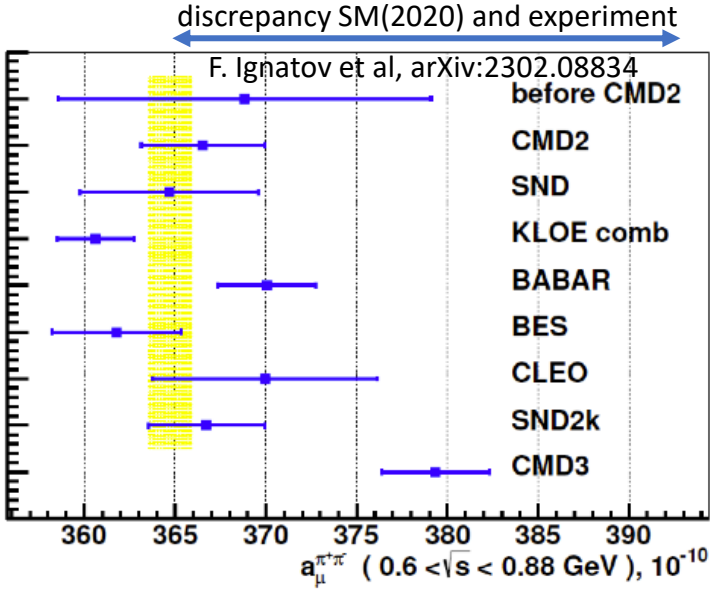


g-2 blinding numbers
 29998956
 39997844
 date 2/28/2018

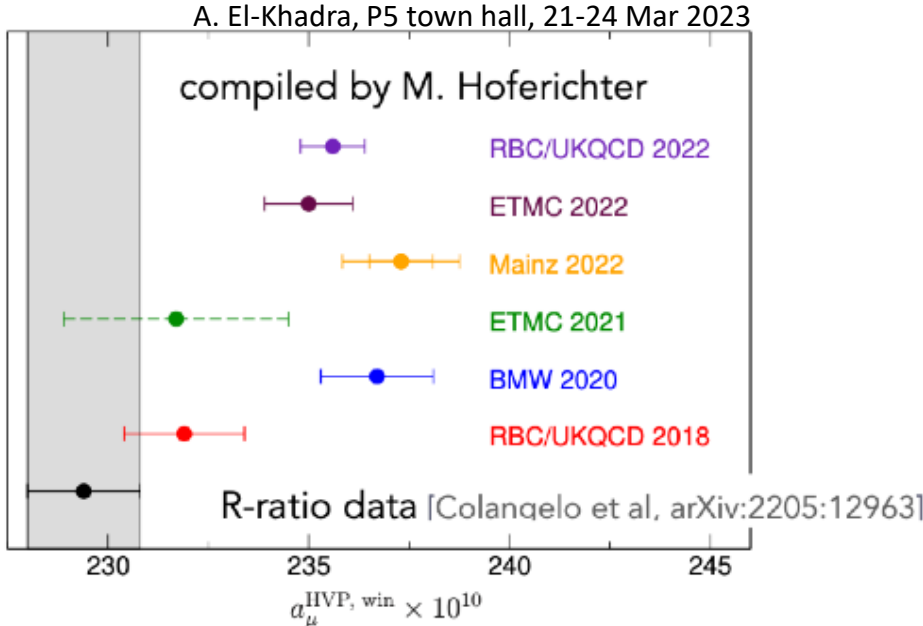
ω_a reference cloc

$$\tilde{\omega}'_p = f_{\text{calib}} \langle M(x, y, \phi) \omega'_p(x, y, \phi) \rangle (1 + B_k + B_q)$$

Dynamic theory situation



A new experimental input to the SM prediction was recently released!



Huge progress in ab-initio calculation of hadronic physics contributions using lattice QCD

Three lepton processes: Naïve scaling

g-2

$$a_l = \frac{g_l - 2}{2} \propto m_l^2$$

EDM

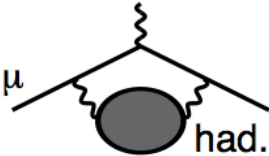
$$d_l \propto m_l$$

LFV

$$\mathcal{B}\nabla(l \rightarrow lX) \propto (m_l)^0$$

Dispersive approach

The diagram to be evaluated:



pQCD not useful. Use the **dispersion relation** and the **optical theorem**.

$$\text{had.} = \int \frac{ds}{\pi(s-q^2)} \text{Im had.}$$

$$2 \text{Im had.} = \sum_{\text{had.}} \int d\Phi \left| \text{had.} \right|^2$$

$$a_{\mu}^{\text{had,LO}} = \frac{m_{\mu}^2}{12\pi^3} \int_{s_{\text{th}}}^{\infty} ds \frac{1}{s} \hat{K}(s) \sigma_{\text{had}}(s)$$

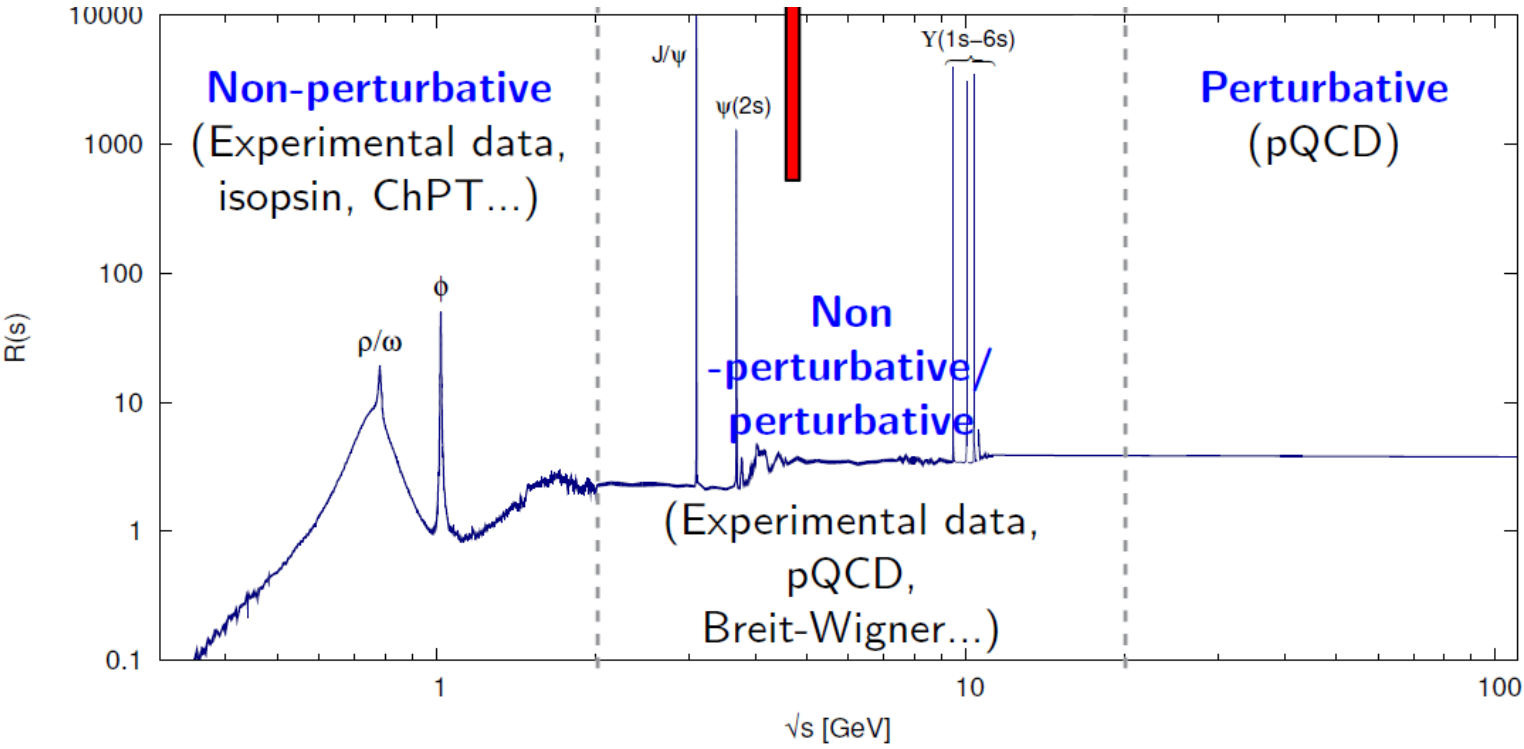
Credit: Thomas Teubner

Follows from causality → analyticity

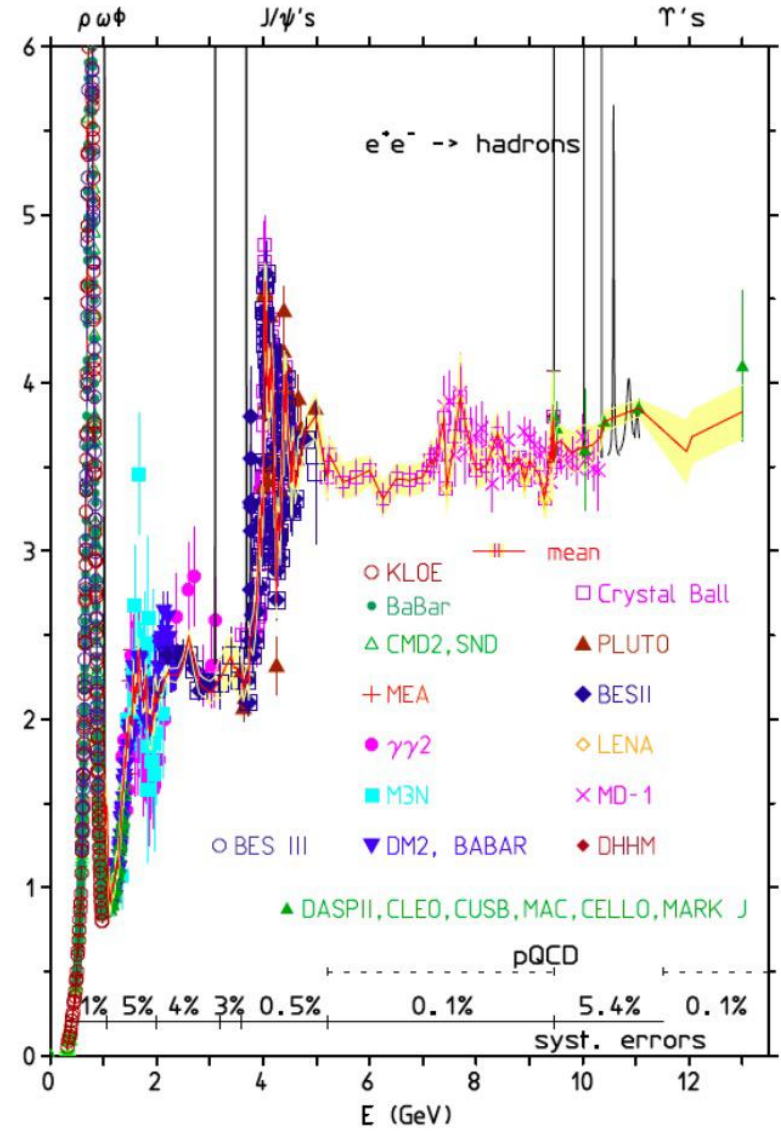
Follows from unitarity of scattering matrix

Weight function $K(s)$ from loop integral $\int d^4q$
 Low energies more important
 $\pi^+ \pi^-$ contribute 73% to LO
 need to know total hadronic cross-section $\sigma_{\text{had}}(s)$

Dispersive approach

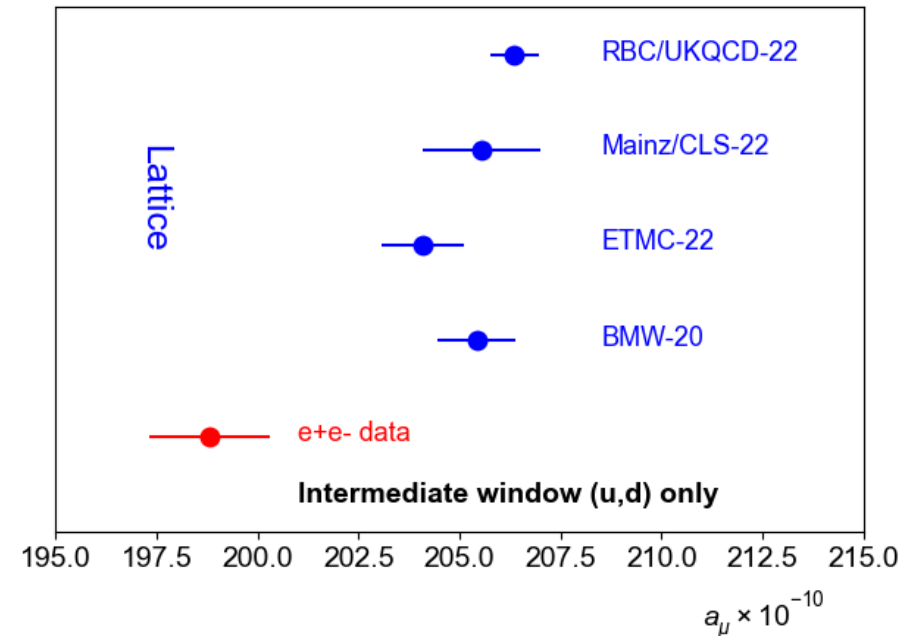
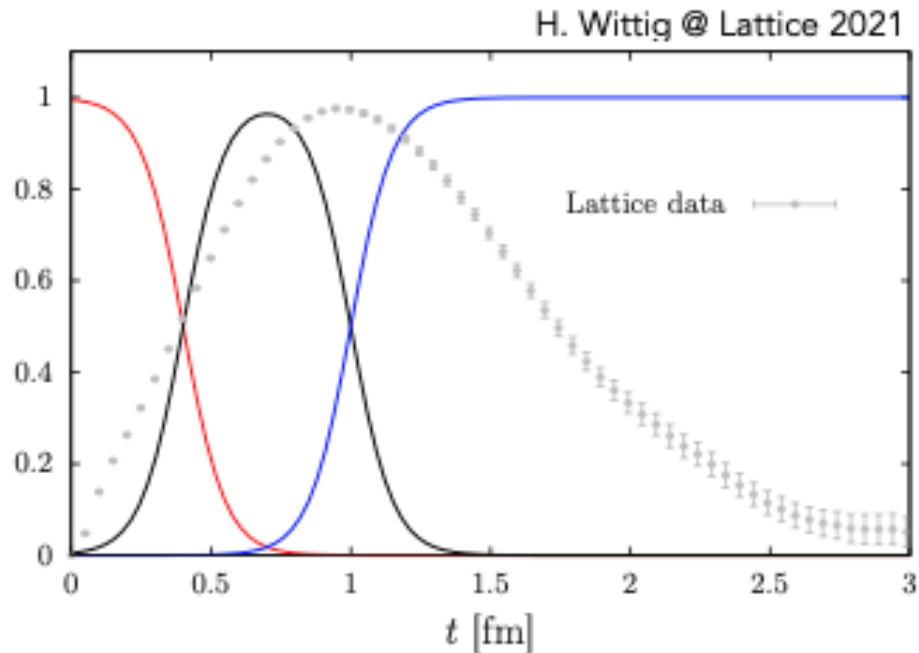


- >100 datasets from $e^+e^- \rightarrow$ hadrons in > 35 final states
- Data from BELLE-II, BES-III, KLOE, BaBar, SND, CMD-3 and KEDR



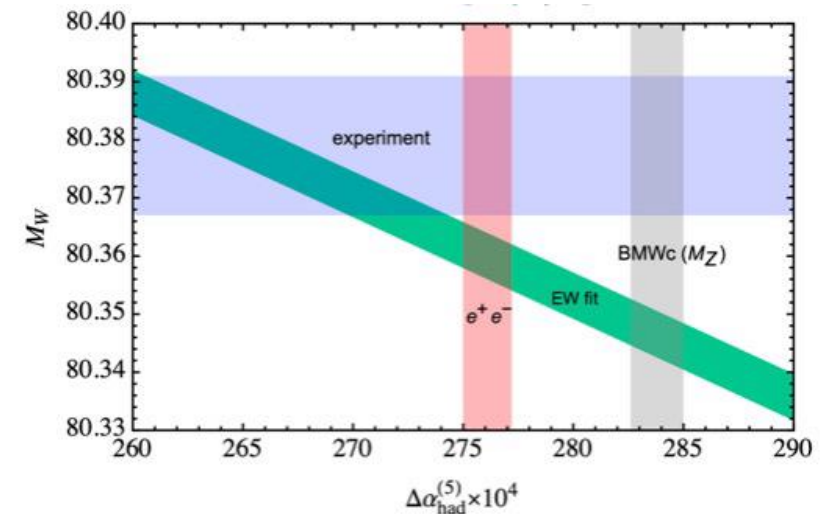
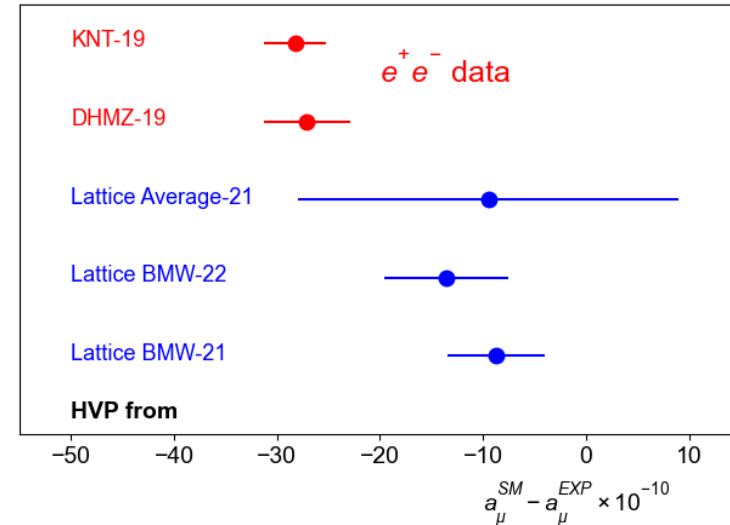
Lattice approach

- First principal calculation by discretizing Euclidian space-time
- BMW is presently the only sub 1% (HVP) lattice calculation in the full kinematic region
- Cross-checks recently performed but only **in limited** (30%) (distance) region.

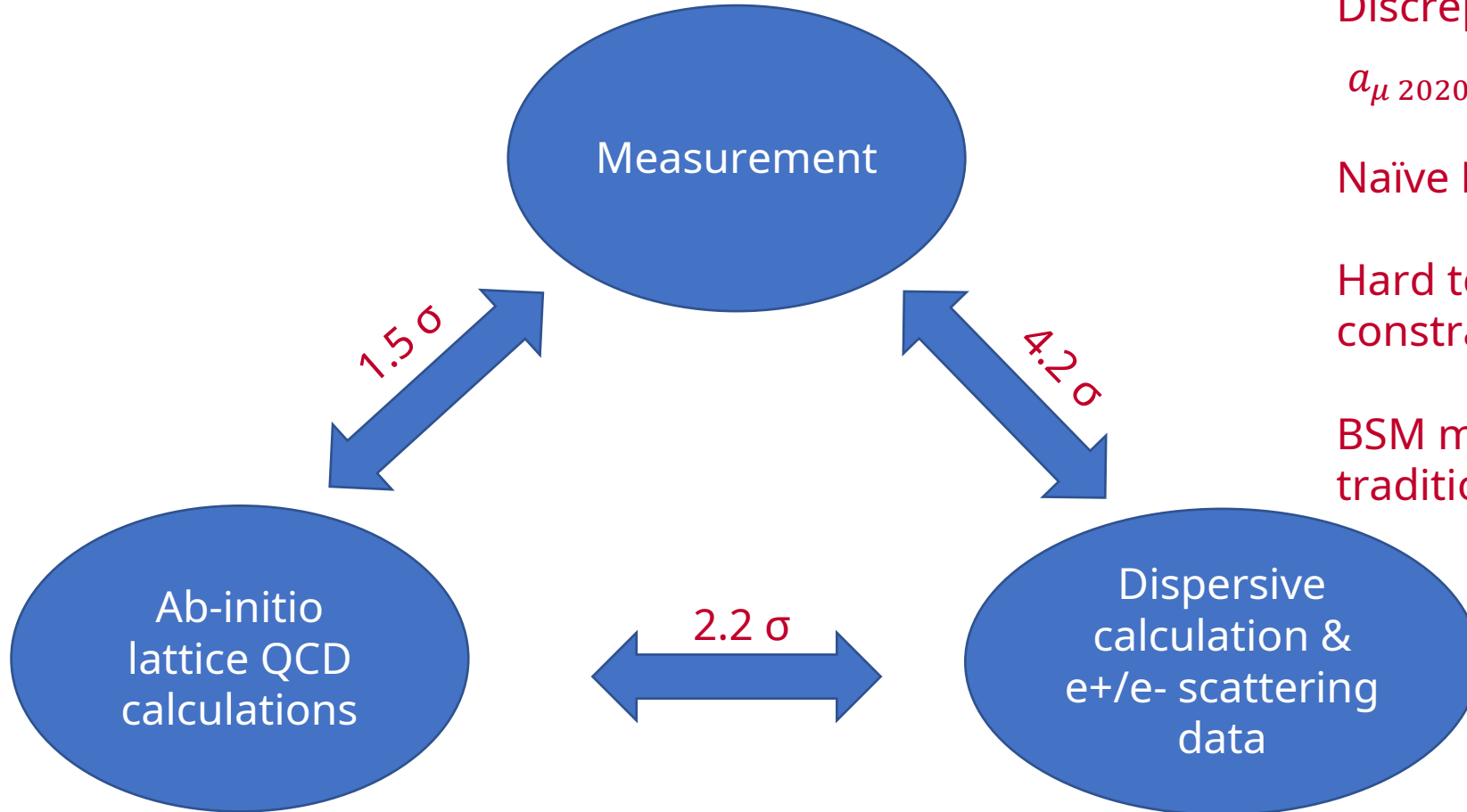


Dispersive vs Lattice Approach

- Implication of BMW results is that there are issues with the e^+e^- measurements (below 0.7 GeV) or a flaw in the e^+e^- or lattice theory
- If this is true then is affected and so are the global EWK fits since they use e^+e^- data
- Tension in SM M_W , M_H vs measured M_W , M_H
- The analysis of e^+e^- data can be made to match the BMW lattice prediction if the measured cross sections below 0.7 GeV **are shifted by 7%**.
- In this region there is data from 9 independent experiments: the most precise experiments (KLOE, BaBar, CMD, SND,) quote **cross section uncertainties of 0.5-1%...**



A new era of a_μ comparisons



Discrepancy is large

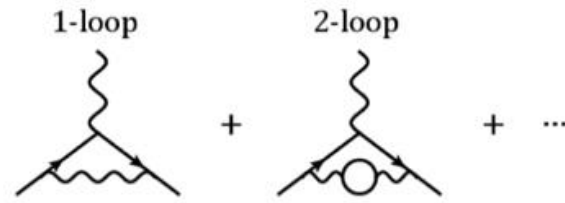
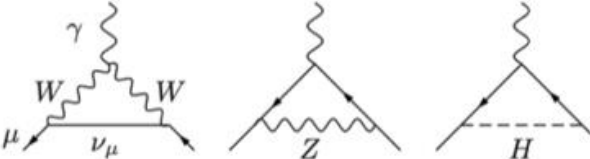
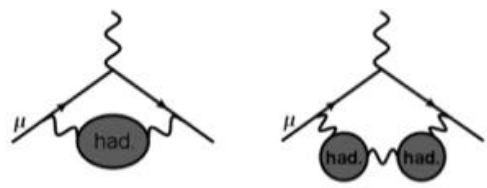
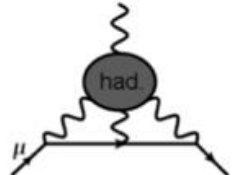
$$a_{\mu 2020} - a_{\mu \text{ dispersive}} \sim 1.7 a_{\mu \text{ Weak}}$$

Naïve NP scaling $M_{BSM} < O(2.1 \text{ TeV})$

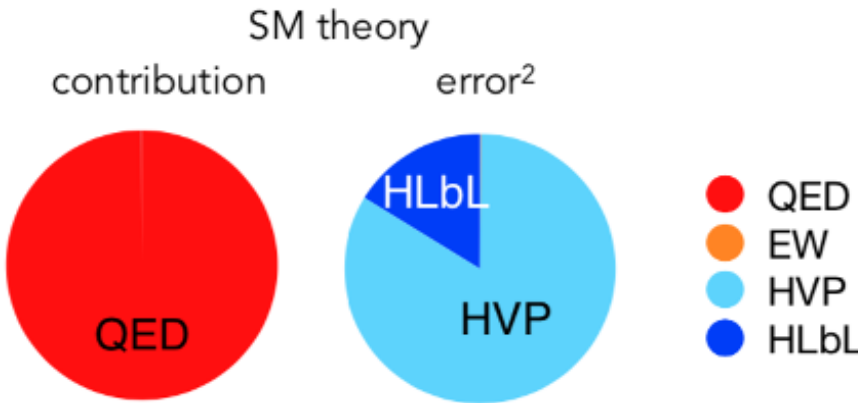
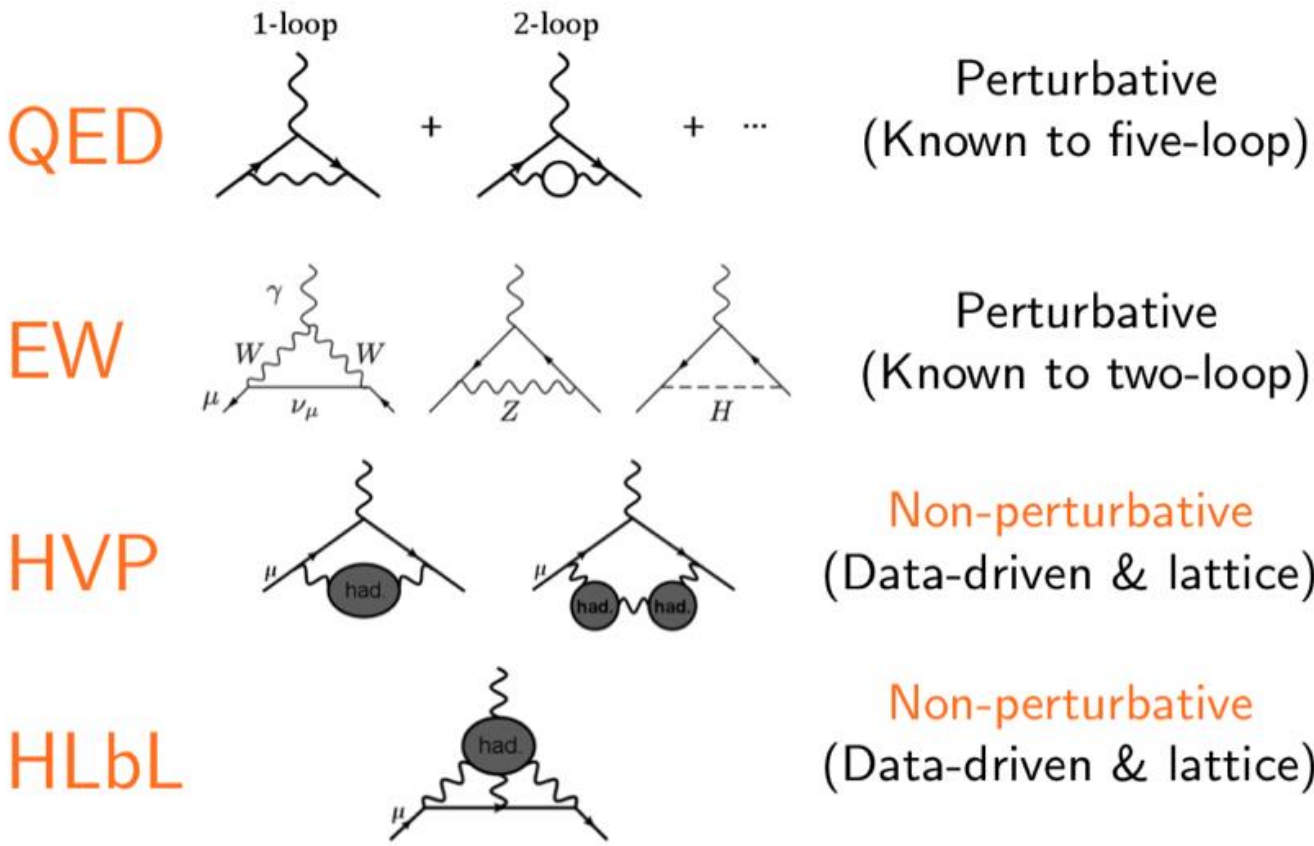
Hard to accommodate with LHC and DM constraints

BSM models tend to be in non-traditional parameter regions

Muon $g-2$ in SM

			a_μ^{SM} portion	δa_μ^{SM} portion
QED	 <p>1-loop + 2-loop + ...</p>	Perturbative (Known to five-loop)	$\sim 99.99\%$	$\sim 0.001\%$
EW	 <p>γ, W, ν_μ, Z, H</p>	Perturbative (Known to two-loop)	~ 1 ppm	$\sim 0.2\%$
HVP	 <p>had, had</p>	Non-perturbative (Data-driven & lattice)	~ 59 ppm	$\sim 84\%$
HLbL	 <p>had</p>	Non-perturbative (Data-driven & lattice)	~ 1 ppm	$\sim 16\%$

Muon $g-2$ in SM



Beyond Standard Model Physics

- Extra contribution to anomalous magnetic moment

$$a_\mu = a_{\text{QED}} + a_{\text{weak}} + a_{\text{hadron}} + a_{\text{BSM}}$$

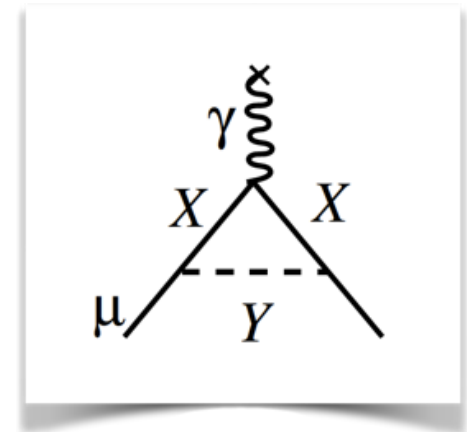
- Naïve scaling

$$\Delta a_1^{\text{BSM}} \propto \frac{g_{\text{BSM}}}{16\pi^2} \frac{(\text{lepton mass})^2}{(\text{new particle mass})^2}$$

- Comparison with electron g-2

$$\left(\frac{m_\mu}{m_e}\right)^2 = \left(\frac{105 \text{ MeV}}{0.5 \text{ MeV}}\right)^2 \approx 43000$$

- Muon g-2 is ~43000 more sensitive to new physics compared to electron g-2

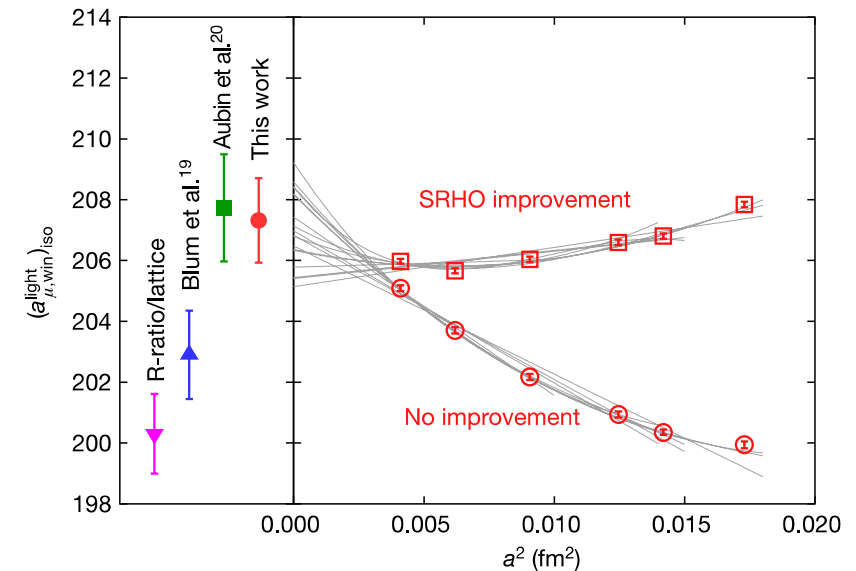
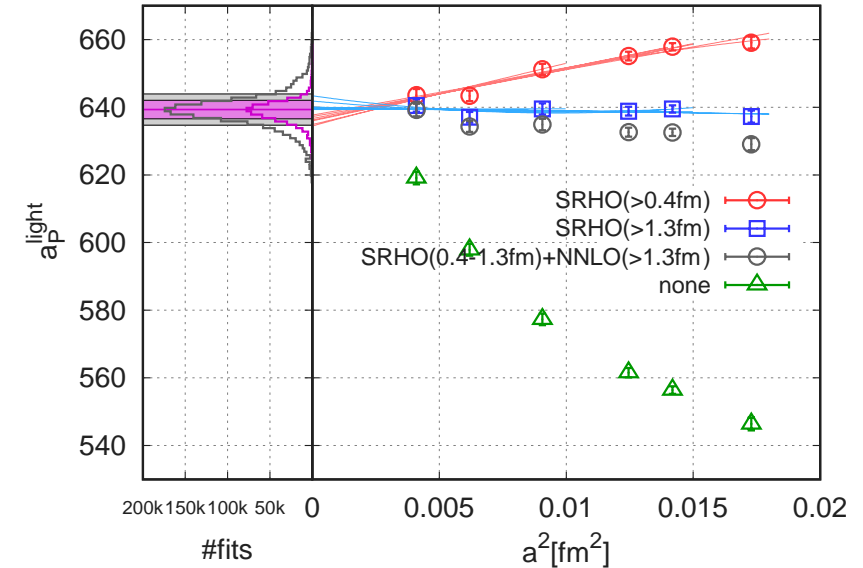


Lattice approach

- BMW20: First sub% calculation of HVP contribution on lattice
- Calculation of “1 particle Irreducible diagrams”

$$\mu \sim \xrightarrow{q} \text{1PI} \sim \nu \equiv i\Pi^{\mu\nu}(q),$$

- Large systematics from **continuum limit**
- upper right panel: limit and uncertainty estimation
- lower right panel: limit for central window compared to other lattice and data-driven results



Lattice approach

